Finnish Mathematical Days 2022

Programme and abstracts

4–5 January 2022

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Online event, 4-5 January 2022

Foreword by the Organizing Committee

Finnish Mathematical Days 2022 is held at Tampere University City Centre Campus in Tampere, Finland, during 4-5 January 2022. It is organized by Finnish Mathematical Society and Tampere University Logic group. The Organizing committee is chaired by Esko Turunen, and its other members are Riikka Kangaslampi, Kerkko Luosto and Jani Hirvonen.

We thank the members of the Program Committee for their effort and time invested in putting together the excellent scientific program for the Conference. We also thank the invited speakers and the speakers of contributed papers for providing the real contents for the Conference.

We are grateful to Johanna Laine and Kristiina Tuoko (TAU Collaboration Services) and the volunteer Zoom moderator group for their help in the practical side of organizing the Conference. For financial support we thank Tampere University and Mathematics Fund.

On the behalf of the Organizing Committee, I wish all the speakers and other participants of Finnish Mathematical Days 2022 welcome to Tampere.

Esko Turunen
Chair of the Organizing Committee

Foreword by the Programme Committee

As with previous editions of Finnish Mathematical Days, the primary aims are to reflect the current activities in mathematics in Finland and to provide a local meeting forum for their communities, broadly conceived. Besides, participation of mathematicians from all over the world are warmly welcomed. The scope of Finnish Mathematical Days 2022 is broad, ranging over all areas of mathematics, from applied to pure mathematics as well as pedagogy of mathematics.

The Program Committee of the Finnish Mathematical Days 2022 consisted of the following members: Pauliina Ilmonen (Aalto University), Maarit Järvenpää (University of Oulu), Tuomo Kuusi (University of Helsinki), Kaisa Miettinen (University of Jyväskylä), Sampsa Pursiainen (Tampere University) and Esko Turunen (Tampere University).

The PC decided to invite six plenary speakers:

Gabriele Eichfelder, Technische Universität Ilmenau

Kaie Kubjas, Aalto University

Daniele Mundici, University of Florence

Daniel Reinholz, San Diego State University

Sara Sommariva, Università di Genova

Ville Vuorinen, Aalto University

Besides, numerous submissions of abstracts for contributed talks were submitted and 75 of them were accepted for presentation at the symposium. We wish to thank all PC members for their contribution to the PC work and all authors of contributed talks for their support.

Esko Turunen
Chair of the Program Committee

Programme

The Finnish Mathematical Days is organised by Tampere University as a remote event via Zoom. The links to the event were sent to registered participants on Monday, January 3rd 2022. There is one Zoom link for each original auditorium, e.g., the invited talks were originally all allocated to auditorium A1.

Tuesday, 4 Jan

8:45–9:00	Opening	
9:00–10:00	Daniele Mundici:	*Computing in AF C^*-algebras via algebraic logic*
10:00–10:30	Break	
10:30–12:00	Parallel sessions A	
12:00–13:30	Lunch break	
13:30–14:30	Kaie Kubjas:	*Algebraic degree of polynomial optimization*
14:40–15:40	Ville Vuorinen:	*Scale-resolved computational fluid dynamics: applications and methods*
15:40–16:00	Break	
16:00–17:30	Parallel sessions B	

Wednesday, 5 Jan

9:00–10:00	Daniel Reinholz:	*Equity analytics: An approach for transforming postsecondary mathematics instruction*
10:00–10:30	Break	
10:30–12:00	Parallel sessions C	
12:00–13:30	Lunch break	
13:30–14:30	Sara Sommariva:	*Data neuroScience: mathematical tools for investigating brain activity from electrophysiological data*
14:40–15:40	Gabriele Eichfelder:	*Direct Methods for Multiobjective Optimization*
15:40–16:00	Break	
16:00–17:30	Parallel sessions D	

Any last minute changes will be updated on the webpage of the meeting.

Invited talks

Daniele Mundici:	*Computing in AF C^*-algebras via algebraic logic*
Kaie Kubjas:	*Algebraic degree of polynomial optimization*
Ville Vuorinen:	*Scale-resolved computational fluid dynamics: applications and methods*
Daniel Reinholz:	*Equity analytics: An approach for transforming postsecondary mathematics instruction*
Sara Sommariva:	*Data neuroScience: mathematical tools for investigating brain activity from electrophysiological data*
Gabriele Eichfelder:	*Direct Methods for Multiobjective Optimization*

Computing in AF C*-algebras via algebraic logic

Daniele Mundici
University of Florence, Italy

An *AF(C*)-algebra* A is the norm-closure of the union of an ascending sequence of finite-dimensional C*-algebras. Elliott proved that the unital dimension (Grothendieck) group $K_0(A)$ uniquely determines A up to isomorphism. AF-algebras with lattice-ordered K_0 (for short, *AFl-algebras*) have a preeminent role in the AF-algebraic literature. Let Γ be the categorical equivalence between unital abelian ℓ-groups and MV-algebras. The Elliott classifier $E(A) = \Gamma(K_0(A))$ of any AFl-algebra A has the structure of a countable MV-algebra. All countable MV-algebras arise as $E(A)$ for some AFl-algebra A. $E(A)$ uniquely determines A up to isomorphism. Elements of $E(A)$ are Murray-von Neumann equivalence classes of projections. Since $E(A)$ is the Lindenbaum algebra of some theory in Łukasiewicz infinite-valued logic Ł$_\infty$, every Ł$_\infty$-formula ϕ naturally codes a Murray-von Neumann equivalence class $[p] = [p]_\phi$ of projections of A. The deductive-algorithmic machinery of Ł$_\infty$ can be applied to decide problems on the projections of A. For instance: Is $[p]$ the class of a central projection of A? Does $[p]$ precede $[q]$ in the Murray-von Neumann order of A? Is $[p] = 0$? The complexity of all these problems turns out to be polytime for many relevant AFl-algebras in the literature, including the Behncke-Leptin algebras $A_{m,n}$, the CAR (Canonical Anticommutation Relation) algebra, the Farey-Stern-Brocot algebra, Glimm's universal UHF algebra, and every Effros-Shen algebra $\mathfrak{F}_\theta$ for θ a real algebraic integer, or $\theta = 1/\mathrm{e}$.

References

[1] G.A. Elliott, On the Classification of Inductive Limits of Sequences of Semisimple Finite-Dimensional Algebras, *J. Algebra*, 38 (1976), 29-44, https://doi.org/10.1016/0021-8693(76)90242-8

[2] D. Mundici, Word problems in Elliott monoids, *Advances in Mathematics*, 335 (2018), 343-371, https://doi.org/10.1016/j.aim.2018.07.015

[3] D. Mundici, Bratteli diagrams via the De Concini-Procesi theorem, *Commun. Contemp. Math.*, 23(7) (2021), 2050073, https://doi.org/10.1142/S021919972050073X

Algebraic degree of polynomial optimization

Kaie Kubjas
Aalto University

We study an optimization problem with the feasible set X defined by polynomials and whose parametric objective function f_u is gradient-solvable with respect to parametric data u. This class includes common objective functions such as the squared Euclidean distance and the log-likelihood function, but also the p-th power of the p-norm for an integer $p \geq 2$.

For Euclidean distance (ED) optimization and maximum likelihood (ML) optimization, a prominent role is played by the ED and ML correspondence, respectively. To our generalized optimization problem we attach an optimization correspondence $\mathcal{F}_X$ which is the set of points $(x, u) \in \mathbb{C}^n \times \mathbb{C}^n$ such that $x \in X$ is critical for f_u. This leads to the notion of algebraic degree of optimization on X. Over general data points $u \in \mathbb{C}^n$, the second projection $\pi_2 : \mathcal{F}_X \to \mathbb{C}^n$ has finite fibers $\pi_2^{-1}(u)$ of constant cardinality. The algebraic degree of X with respect to the objective function f_u is the cardinality of the fiber $\pi_2^{-1}(u)$ over a general data point u. The algebraic degree of optimization measures the algebraic complexity of the optimal solution. We apply these results to p-norm optimization, and define the p-norm distance degree of X, which coincides with the ED degree of X for $p = 2$.

This talk is based on joint work with Olga Kuznetsova and Luca Sodomaco.

Scale-resolved computational fluid dynamics: applications and methods

Ville Vuorinen
Aalto University

Computational fluid dynamics (CFD) offers a methodological framework for simulating 3D air and liquid flows. During the past two decades, high performance computing has enabled usage of scale-resolving methods to simulate turbulent fluid flows. In such chaotically swirling flows common to many real-life applications, the non-linear interactions of the Navier-Stokes equations lead to multiscale physics. To capture such physical phenomena, numerical solution of the partial differential equations on a fine space-time resolution is required.

In the first part of the talk, I will discuss the fundamental methods to solve the Navier-Stokes equation. In the second part of the talk I focus on recent application of finite difference/Fourier methods to simulate airflow and transmission of airborne viruses in indoor environments on a GPU. Finally, In the third part of the talk I discuss a few applications of fluid dynamics including chemically reactive flow and two phase flow using an open source finite volume code.

Equity analytics: An approach for transforming postsecondary mathematics instruction

Daniel L. Reinholz
San Diego State University, United States

This talk introduces the EQUIP observation tool (https://www.equip.ninja) and describes the equity analytics approach to improving mathematics instruction. While mathematics education research provides insight into a variety of new instructional practices, supporting the equitable use of such practices in postsecondary mathematics has remained a challenge. One of the key barriers is that instructors lack practical data about what is happening in their classrooms. The equity analytics approach overcomes this barrier by providing instructors with actionable data that they can use to refine their teaching in an iterative fashion.

In this talk, I draw upon multiple case studies of mathematics faculty members and share the details of the educational transformations they have achieved with the support of equity analytics and EQUIP-based coaching. These cases provide models for what other instructors can hope to achieve in their classrooms. In addition, I provide details of the EQUIP approach so that others can use it for self-study or in coaching contexts.

Data neuroscience: mathematical tools for investigating brain activity from electrophysiological data

Sara Sommariva
University of Genoa, Italy

Most brain functions are regulated by tiny intercorrelated electrical currents flowing in few specific brain areas. Magneto- and electro-encephalography (M/EEG) are two modern neuroimaging techniques capable of non-invasively recording the electromagnetic field produced outside the scalp by these neural currents. Interpreting the recorded M/EEG data is not straightforward, due for example to source-leakage effects induced by volume conduction, and advanced mathematical techniques are required to estimate the dynamical brain activity that has generated the measured data. A typical workflow of analysis consists of two steps: (i) first the active brain regions and their time-courses are estimated by solving and ill-posed inverse problem (ii) then a proper connectivity metric is compute to quantify the statistical dependencies between the time-courses estimated at different brain locations.

The aim of this talk is to provide some insights on modern data-science tools to automatically estimate brain activity and connectivity from M/EEG data.

In detail, in the first part of the talk I will formulate the M/EEG inverse problem within a Bayesian setting and I will present SESAMEEG (SEquential Semi-Analytic Montecarlo Estimation for MEEG [1]) a Sequential Monte Carlo sampler we developed to obtain fully-automatic parametric representations of brain activity.

In the second part of the talk I will focus on brain connectivity. Some recent studies have demonstrated that the classical two step approach previously described may be suboptimal when Tikhonov regularization is used to solve the M/EEG inverse problems. Specifically, unexpected issues in setting the regularization parameter may arise [2]. Motivated by these results I will show how to derive an inverse problem that allows to directly estimate the cross-power spectrum of the neural sources from that of the recorded data. Then I will present a lasso-based approach we developed to obtain sparse estimates of the brain functional networks.

References

[1] S. Sommariva and A. Sorrentino, Sequential Monte Carlo samplers for semi-linear inverse problems and application to magnetoencephalography, *Inverse Problems* 30(11) (2014), 114020.

[2] E. Vallarino, S. Sommariva, M. Piana, and A. Sorrentino, On the two-step estimation of the cross-power spectrum for dynamical linear inverse problems, *Inverse Problems* 36(4) (2020), 045010.

Direct Methods for Multiobjective Optimization

Gabriele Eichfelder
Ilmenau University of Technology, Germany

In multiobjective optimization one considers optimization problems with several competing objective functions. For instance, in engineering, a design often has to be stable and light at the same time. A classical approach to such optimization problems is to formulate suitable parameter-dependent single-objective replacement problems, called scalarizations, such as a weighted sum of the objective functions or the epsilon-constraint scalarization. Then, the parameters are varied and the scalarized problems are solved iteratively.

However, many classes of multiobjective optimization problems have a structure where a scalarization might not be the best approach for an efficient procedure. In this talk, we introduce the basic concepts and some of the classical solution approaches in multiobjective optimization. Thereby, we cover the basic ideas of scalarization-based methods. Then, we present examples of classes of multiobjective optimization problems for which it might be better not to scalarize the problems first. These classes include non-convex continuous problems, convex mixed integer and integer problems, and heterogeneous problems, where one of the objective functions is assumed to be an expensive black-box function while the other objectives are analytically given. We give some of the main ideas behind other solution approaches for them.

Parallel sessions

The talks of the parallel sessions start according to the following table, unless the chair indicates otherwise in the beginning of the session.

	sessions	**1st talk**	**2nd talk**	**3rd talk**
A	1–7	10:30	11:00	11:30
B	8–14, 27	16:00	16:30	17:00
C	15–21	10:30	11:00	11:30
D	22–26, 28, 29	16:00	16:30	17:00

Parallel sessions A

1 Transcendental number theory (chair: Anne-Maria Ernvall-Hytönen)

Topi Törmä: *Generalized continued fraction Cantor sets*
Neea Palojärvi: *On a p-adic lower bound for a linear form in logarithms*
Tapani Matala-aho: *Euler's factorial series, Hardy integral, and continued fractions*

2 Statistics (chair: Sara Taskinen)

Santeri Karppinen: *Conditional particle filters with diffuse initial distributions*
Jarkko Isotalo: *Prediction and Testing of Random Effects in Linear Mixed Models*
Joni Virta: *Subsphericity testing in high dimensions*

3 Functional Analysis 1 (chair: Hans-Olav Tylli)

Jani Virtanen: *Fredholm theory of Toeplitz operators on doubling Fock spaces*
Santeri Miihkinen: *On the exponential integrability of conjugate functions*
Antti Perälä: *Integration operators from Bergman spaces to Hardy spaces of the unit ball*

4 Inverting Electromagnetic Fields in Complex Domains: Brain and Asteroids (chair: Sampsa Pursiainen)

Atena Rezaei: *Applying Inverse modeling techniques to detect simultaneous subcortical and cortical brain activity: Targeting reconstruction of SEP components*
Yusuf Yusuf: *Full-Wave Radar Tomography: an Application to Small Solar System Bodies in the Presence of Wavelength-induced Errors*
Elisabetta Vallarino: *Two-step estimation of source-space brain connectivity from MEG data: insights on the choice of the regularization parameter*
Alessandro Viani: *Bayes strikes back: free hyper-parameter selection and averaging by SMC samplers in NEF distributed likelihood inverse problems*

5 Yliopistomatematiikan etä- ja hybridiopetus (chair: Terhi Kaarakka)

Matti Pauna: *Raja-arvotehtävien toteuttamista Stackilla*
Juha-Matti Huusko: *JSXGraph-havainnollistuksia matematiikasta ja elämästä*
Antti Valmari: *Joustavan matemaattisen päättelyn automaattinen tarkastaminen lineaaristen (itseisarvo)(epä)yhtälö(ryhmien) tapauksessa*

6 Industrial Mathematics (chair: Marko Vauhkonen)

Tuomo Kauranne: *Digital twins of forest ecosystems*
Sebastian Springer: *Sawing Optimization, a Tandem Forest values project*
Jaakko Pihlajasalo: *Deep Learning Based OFDM Physical-Layer Receiver for Extreme Mobility*

7 Mathematical logic 1 (chair: Åsa Hirvonen)

Fan Yang: *Intermediate logics in the team semantics setting*
Ulla Karhumäki: *On Cherlin–Zilber Algebraicity conjecture*
Joni Puljujärvi: *Independence Logic and Quantum Physics*

Generalized continued fraction Cantor sets

Topi Törmä

By a generalized continued fraction Cantor set we mean the set

$$E_{\mathcal{B}}^{\mathcal{A}} = \left\{ \frac{a_1}{b_1 + \dfrac{a_2}{b_2 + \cdots}} : a_n \in \mathcal{A}, b_n \in \mathcal{B} \text{ for all } n \right\},$$

where $\mathcal{A}$ and $\mathcal{B}$ are some given finite sets of positive integers. We give an overview of what is known about the possible asymptotic irrationality exponents of numbers in $E_{\mathcal{B}}^{\mathcal{A}}$. We also consider when the set $E_{\mathcal{B}}^{\mathcal{A}}$ contains an interval.

On a p-adic lower bound for a linear form in logarithms

Neea Palojärvi, University of Helsinki

Linear forms in logarithms can be used to find bounds for solutions of certain Diophantine equations. Further, lower bounds for linear forms are also related to measures of irrationality and transcendence.

In this talk, I will discuss about an explicit lower bound for the p-adic absolute value of a linear form in p-adic logarithms of given rational numbers. The lower bound is obtained using Padé approximations. This is joint work with Louna Seppälä.

Euler's factorial series, Hardy integral, and continued fractions

Tapani Matala-aho, Aalto University

Let p be a prime and let

$$E_p(t) = \sum_{k=0}^{\infty} k! t^k$$

denote the Euler's factorial series. We will present recent results on lower bounds for the p-adic absolute value of the expression $dE_p(p^a) - c$, where $a, c, d \in Z$. The proofs are based on the fact that the same Padé polynomials which p-adically converge to $E_p(t)$, approach the Hardy integral $\mathcal{H}(t) = \int_0^\infty \frac{e^{-s}}{1-ts} \mathrm{s}$ on the Archimedean side. Furthermore, we will discuss on an interconnection between $E(t)$ and $\mathcal{H}(t)$ via continued fractions.

The results are based on joint works with Anne-Maria Ernvall-Hytönen, Louna Seppälä and Wadim Zudilin.

Conditional particle filters with diffuse initial distributions

Santeri Karppinen
Department of Mathematics and Statistics
University of Jyväskylä

Conditional particle filters (CPFs) and their backward sampling variants are powerful Markov chain Monte Carlo algorithms for generic nonlinear and/or non-Gaussian state space model smoothing. However, CPFs can be inefficient or difficult to apply with diffuse (mildly informative or uninformative) initial distributions, which are common in statistical applications. In this presentation, I discuss a simple and generally applicable auxiliary variable method which can be used together with the CPF to perform efficient statistical inference for generic state space models with diffuse initial distributions. The method only requires simulatable Markov transitions that are reversible with respect to the initial distribution of the state space model, which may also be improper. To facilitate efficient mixing, an online adaptation scheme may also be embedded within our method. This talk will review the CPF and then discuss our method as an extension of it. Results of applying the method with a stochastic SEIR modelling the COVID-19 pandemic with varying transmission rate are shown. The talk is based on the paper: Karppinen, S., Vihola, M. Conditional particle filters with diffuse initial distributions. Stat Comput 31, 24 (2021). https://doi.org/10.1007/s11222-020-09975-1

Prediction and Testing of Random Effects in Linear Mixed Models

Jarkko Isotalo

We consider the linear mixed effects model

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{u} + \boldsymbol{\varepsilon},$$

where $\mathbf{y}$ is an $n \times 1$ observable random vector, $\mathbf{X}$ and $\mathbf{Z}$ are known model matrices, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown fixed parameters, $\mathbf{u}$ is a $q \times 1$ vector of unobservable random effects, and $\boldsymbol{\varepsilon}$ is an $n \times 1$ unobservable random error vector. We further assume that random vectors $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ are uncorrelated and are normally distributed, i.e., $\mathbf{u} \sim N(\mathbf{0}, \sigma^2\mathbf{G}_{\boldsymbol{\theta}}), \boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma^2\mathbf{R}), \mathrm{Cov}(\mathbf{u}, \boldsymbol{\varepsilon}) = \mathbf{0}$, where σ^2 is a positive unknown scalar and $\boldsymbol{\theta}$ is an unknown parameter vector.

In this setup, we consider different methods for predicting the conditional mean $\mathbf{x}_*'\boldsymbol{\beta} + \mathbf{z}_*'\mathbf{u}$, when $\mathbf{x}_*$ and $\mathbf{z}_*$ are known given vectors. Particularly, we give conditions when the BLUP of $\mathbf{x}_*'\boldsymbol{\beta} + \mathbf{z}_*'\mathbf{u}$ is equal to the BLUE of the conditional mean, see [1]. Obtained results on prediction are then applied to the problem of testing hypotheses set on the conditional mean. In linear mixed models, hypothesis testing related to the random effects are often done by defining the structure of the covariance matrix $\mathbf{G}_{\boldsymbol{\theta}}$ in competing hypotheses and then testing them by using the likelihood ratio statistic or by some other suitable test statistic, see, e.g., [2]. In this research, we set hypotheses on the conditional mean $\mathbf{x}_*'\boldsymbol{\beta} + \mathbf{z}_*'\mathbf{u}$ and then consider different approaches for testing them. Specifically, we show that testing statistic obtained by the interval prediction with use of the BLUP is equivalent to the extended likelihood ratio test statistic.

References:

[1] Puntanen, S., Isotalo, J. & Haslett, S. J., (2021). Equalities between the BLUEs and BLUPs under the partitioned linear fixed model and the corresponding mixed model, *Acta et Commentationes Universitatis Tartuensis de Mathematica*, 5, 2, 239-257.

[2] Hui, F.K.C., Muller, S.M. & Welsh, A.H., (2019). Testing random effects in linear mixed models: another look at the F-test (with discussion), *Australian and New Zealand Journal of Statistics*. 61(1), 61-84.

Subsphericity testing in high dimensions

Joni Virta, University of Turku

Subsphericity testing is encountered most commonly when determining an appropriate number of principal components in PCA. In this work we review a particular test of subsphericity and investigate its asymptotic behaviour in both low- (fixed p) and high-dimensional ($p \equiv p_n$ grows with n) regimes. The test is shown to provide consistent estimates of the latent dimension under a particular, practically intuitive high-dimensional model.

Fredholm theory of Toeplitz operators on doubling Fock spaces

Jani Virtanen, University of Reading and University of Helsinki

Denote by F^2 the Fock space of Gaussian square-integrable entire functions on $\mathbb{C}^n$ and define the Toeplitz operator T_f on F^2 with symbol f by $T_f g = P(fg)$. In the late 1980s, Berger and Coburn characterized the Fredholm properties of T_f for bounded symbols f of vanishing mean oscillation using C^*-algebra techniques, Hilbert space methods and other related heavy machinery. Similar characterizations can be obtained for Toeplitz operators acting on weighted Fock spaces F^p_φ of entire functions in $L^p(\mathbb{C}^n, e^{-p\varphi}dv)$, where dv is the usual Lebesgue measure on $\mathbb{C}^n$. Indeed, Berger and Coburn's characterization was recently extended to standard Fock spaces F^p_α with $1 < p < \infty$ and $\varphi(z) = \alpha|z|^2$ using more elementary methods by Al-Qabani and the author and using novel limit operator techniques by Fulsche and Hagger. This talk explains how similar characterizations can be obtained for unbounded symbols, small exponents ($0 < p \leq 1$), and more general weights. Of particular interest are the weights for which the Laplacian $\Delta\varphi$ of φ is bounded below and above or when $\Delta\varphi$ is a doubling measure. Joint work with Zhangjian Hu.

On the exponential integrability of conjugate functions

Santeri Miihkinen
Karlstad University

The conjugate function $\tilde{f}$ (the periodic Hilbert transform) of an integrable function $f\colon \mathbb{T} \to \mathbb{R}$ on the unit circle $\mathbb{T}$ can be defined as the principal value integral

$$\tilde{f}(\theta) = \lim_{\epsilon \to 0} \frac{1}{2\pi} \int_{|\theta-\varphi|>\epsilon} \cot\left(\frac{\theta-\varphi}{2}\right) f(\varphi)\, d\varphi$$

for almost every θ.

Comparison of sizes of f and $\tilde{f}$ in different function spaces is a topic of significant interest in the literature. Although $f \in L^\infty$ does not imply that $\tilde{f} \in L^\infty$, the conjugation operator $f \mapsto \tilde{f}$ still has very strong boundedness properties. Namely, a classical theorem of Zygmund asserts that if $\|f\|_{L^\infty(\mathbb{T})} \leq \frac{\pi}{2}$ and $\lambda < 1$, then $\exp(\lambda \tilde{f})$ is integrable.

We investigate exponential non-integrability of conjugate functions, i.e., the reverse direction to Zygmund's result. The talk is based on a joint work with Hussain Gissy and Jani Virtanen (University of Reading).

Integration operators from Bergman spaces to Hardy spaces of the unit ball

Antti Perälä, Umeå University

We completely characterize the boundedness of the Volterra type integration operators acting from the weighted Bergman spaces to the Hardy spaces of the unit ball. A partial solution to this problem in the one-dimensional setting was previously obtained by Zhijian Wu. We solve the missing cases and extend the results to all dimensions. Our tools involve area methods from harmonic analysis, Carleson measures and Kahane-Khinchine type inequalities, as well as techniques and integral estimates related to Hardy and Bergman spaces. This talk is based on a joint paper with Santeri Miihkinen, Jordi Pau and Maofa Wang.

Applying Inverse modeling techniques to detect simultaneous subcortical and cortical brain activity: Targeting reconstruction of SEP components

Atena Rezaei[1], Joonas Lahtinen[1], Frank Neugebauer[1,2], Marios Antonakakis[2], MariaCarla Piastra[3], Alexandra Koulouri[1], Carsten H. Wolters[2], and Sampsa Pursiainen[1]

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In this study, we investigate reconstructing the sequential components of somatosensory evoked potentials (SEPs) at different latencies between 14-30 ms. Detecting subcortical activity is a challenging task since subcortical structures are far from the sensors on the scalp, however, it plays a significant role in treatment of brain disorders such as Alzeihemr's disease or refractory epilepsy. To this end, we applied randomized multiresolution scanning (RAMUS) technique in the framework of hierarchical Bayesian, conditionally Gaussian approach which is recently proposed as a potential technique to reconstruct cortical and simultaneous subcortical activity. Our goal is to detect the simultaneous cortical and weak deep activity in reconstructing the generator of SEP components of median nerve stimulation for earlier and late components. For our analysis, we used three different realistic head models and experimental measurement datasets. RAMUS is a maximum a posterior estimation technique aiming to reduce optimization and discretization errors via randomized source spaces and coarse to fine reconstruction strategy. The source space is decomposed into random subsets during a refinement process which is repeated for several randomized configurations and the final reconstruction is obtained as an average over the multiple resolution levels. RAMUS found weakly distinguishable deep activity for earlier components while simultaneous cortical and subcortical activity was detected for components after 20 ms. The earliest SEP component, P14/N14, was reconstructed at medial lemniscus and SEP component, P20/N20, was localized at Brodmann area 3b. In order to compare the performance of RAMUS, we applied other inversion techniques such as sLORETA and Beamformer for reconstructing the SEP components at aforementioned latencies. Our findings reveal RAMUS as a promising method for localizing cortical and weakly distinguishable deep activity in the case of realistic and multicompartment head models and experimental measurements.

Full-Wave Radar Tomography: an Application to Small Solar System Bodies in the Presence of Wavelength-induced Errors

Yusuf Oluwatoki Yusuf[1,2], Astrid Dufaure[2], Liisa-Ida Sorsa[1], Christelle Eyraud[2], and Sampsa Pursiainen[1]

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[2]Aix-Marseille Univ., CNRS, Centrale Marseille, Institut Fresnel, Marseille, France

There are over $27,313$ known NEAs, i.e., within 1.945×10^8 km from the sun and only 4.488×10^7 km from Earth, of which $1,254$ are on the risk list of ESA's Near Earth Object Coordination Centre (NEOCC). Hence, advancing the knowledge of mineral composition, interior structure, and potential threats of planetary bodies is very important which has been the goal of space explorations over years. The first space mission to a small Solar System Body (sSSB), aimed at preforming Radar Tomography (RT) was Comet Nucleus Sounding Experiment by Radio Transmission (CONSERT) by European Space Agency's (ESA's) Rosetta mission in 2004. ESA will continue RT exploration as a part of the HERA mission [2, 3], the European component of AIDA (Asteroid Impact and Deflection Assessment), whose Juventas Radar (JuRa) [1] carried by a CubeSat with the same name (Juventas CubeSat), will perform RT measurements with the asteroid moon of the binary asteroid Didymos as its target. Reconstructing an SSSB's interior permittivity distribution (structure) via RT is an ill-posed inverse problem and utilising the full-waveform approach, i.e., maintaining the complete wavefield information of the forward simulation, is essential with respect to the reconstruction quality.

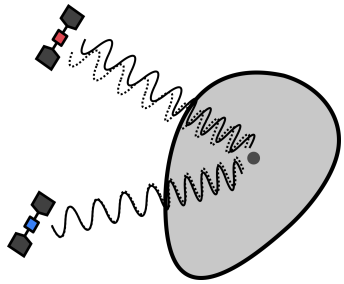

Figure 1: Modelling errors as a result of phase discrepancy leading to large random fluctuations in the demodulated baseband data.

We aim to investigate the interior structure of a two-dimensional test object via numerically simulated full-wave time domain radar tomography with the presence of carrier wave induced uncertainty, hence formulating a statistical model to marginalise such uncertainty. This will be the focus of my discussion at the Finnish Mathematics days 2022 for the session on Inverting Electromagnetic Fields in Complex Domains: Brain and Asteroids.

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Two-step estimation of source-space brain connectivity from MEG data: insights on the choice of the regularization parameter

Elisabetta Vallarino
Università degli Studi di Genova

Source-space brain functional connectivity aims at quantifying and explaining the statistical dependencies that can occur between different brain regions. Thanks to its temporal resolution, magnetoencephalography (MEG) is providing a crucial contribution to the investigation of brain connectivity by non-invasively recording time-courses of measurements associated to the magnetic field induced by neural currents. Connectivity estimation is usually achieved in a two–step process: (i) first the neural time-courses are estimated by solving the MEG inverse problem, (ii) then connectivity is computed by means of a proper metric. Thus, when using Tikhonov regularization as inverse method, the parameter that is set to estimate the time-courses is used throughout the whole process. However, such a parameter is suboptimal for connectivity estimation. Indeed, in previous works, we demonstrated that a smaller parameter should be set for the latter intent, when using the cross-power spectrum as connectivity metric [1, 2].

In this talk, I will show that such a counter-intuitive behaviour of the regularization parameters applies also to a wide range of connectivity metrics, other than the cross-power spectrum. Specifically, we used Thikonov regularization as inverse method, and we quantified connectivity through different metrics, namely the imaginary part of coherence, the weighted phase locking value and the corrected imaginary phase lag index. We defined the optimal parameter for source estimation ($\lambda_{\mathbf{x}}$) as the value minimizing the ℓ_2-norm of the difference between actual and estimated time-courses; and the optimal parameter for connectivity estimation ($\lambda_{\mathbf{C}}$) as the value maximizing the AUC (i.e the area under the receiving operating characteristic (AUC) curve). We simulated several sensor-level data generated from as many configurations of brain activity and we numerically computed the optimal parameters.

Finally, I will also show some preliminary results on a possible one-step approach for connectivity estimation, where the cross-power spectrum at source level is directly estimated from that at sensor level.

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Bayes strikes back: free hyper-parameter selection and averaging by SMC samplers in NEF distributed likelihood inverse problems

Alessandro Viani, Alberto Sorrentino,
Math department, University of Genova, Italy

We present an innovative method for free hyper-parameter selection and averaging by the usage of Sequential Monte Carlo (SMC) samplers in Bayesian inverse problems.

SMC samplers are particular Monte Carlo Methods based on the sequential application of particle selection and density mutations, considering a sequence of more and more complex densities; where the first is easy to sample, and the last one represents the target density.

We show how to obtain an SMC sampler where each density of the sequence can be interpreted as a posterior distribution conditioned on a different value for an hyper-parameter under the assumption of a Natural Exponential Family (NEF) distributed likelihood. For this class of inverse problems this approach permit to wisely select a particular value for the hyper-parameter or its marginalization for free, and at the same time to recycle the entire SMC samplers particles.

The most straightforward application is the approximation of the posterior distribution for both the unknowns and the hyper-parameter in hierarchical Bayesian inverse problems, such as in source analysis of Magneto/Electro-Encephalography (MEG) data, where the hyper-parameter is represented by the noise standard deviation.

For analyzing the performances of the algorithm we provide a toy example of a conditionally linear Gaussian problem; showing that the proposed approach is substantially better than the most common one where the hyper-parameter is simply sampled by the SMC sampler, increasing computational costs.

Concluding, we show that the proposed approach provides good results and at the same time a significant reduction of computational cost compared with different approaches.

Raja-arvotehtävien toteuttamista Stackilla

Matti Pauna, Helsingin yliopisto

Yliopistotasoisen matemaattisen analyysin opetuksessa harjoitellaan lukujonojen raja-arvotulosten täsmällistä todistamista reaalilukujen täydellisyysominaisuuteen perustuvalla nk. epsilon-delta tekniikalla. Sen sisäistäminen on monelle juuri lukiosta tulleelle opiskelijalle sekä käsitteellisesti että teknisesti haasteellista. Todistuksen lähtökohtana tarkastellaan itseisarvoepäyhtälöä tavalla, jota lukion matematiikan opiskelussa ei välttämättä ole kohdattu.

Tämän haasteen ylipääsemiseksi opiskelijaa voi auttaa todistustekniikan harjoittelu omassa tahdissa niin, että on mahdollista saada mielekästä palautetta todistuksen eri kohdissa. Stack on automaattisesti tarkistettavien matematiikan tehtävien toteutukseen sopiva työkalu, jolla tällaista todistusprosessia voidaan simuloida.

Esittelen Helsingin yliopistossa käynnistetyn työn tuloksena syntyneitä raja-arvoihin liittyviä tehtäviä. Lähtökohtaisesti tehtävässä voidaan esimerkiksi yksinkertaistaa matemaattista lauseketta niin, että sille löydetään numeeriset ala- ja ylärajat. Esimerkiksi on määrättävä luvut a, b, c ja d niin, että epäyhtälö

$$\frac{a}{b} \leq \frac{5n+1}{n+5} \leq \frac{c}{d}$$

pätee kaikilla positiivisilla kokonaisluvuilla n. Opiskelija voi Stack-tehtävän avulla määrittää alarajan arvioimalla vaiheittain osoittajaa alaspäin ja nimittäjää ylöspäin kunnes sopivat luvut a ja b löytyvät.

Lukujonon raja-arvoa selvitettäessä pitää annetulle mielivaltaiselle positiiviselle luvulle ε löytää jokin indeksi K siten, että jonon termit ovat lähempänä kuin ε halutusta raja-arvosta indeksistä K alkaen. Jos käytetään rationaalilausekkeita jonon termeinä, ylläolevasta arviointitekniikasta on hyötyä sopivan indeksin löytämisessä. Stackilla on mahdollista laatia tehtäviä, joilla tämän tyyppistä prosessia voi harjoitella tietokoneen avulla syöttämällä arvioinnin välivaiheita ja lopuksi syöttämällä arvo indeksille K, sekä lisäksi saada mielekästä palautetta vastauksista.

JSXGraph-havainnollistuksia matematiikasta ja elämästä

Juha-Matti Huusko

JSXGraph on Javascript-kirjasto, jonka avulla voi tehdä dynaamisia kuvia verkkosivuille. Lopputulos muistuttaa GeoGebralla tehtyjä kuvia.

Kun käytetään Javascript-ohjelmointikielen ominaisuuksia, kuvien tekeminen helpottuu ja monipuolistuu. Pisteitä voi generoida for-silmukan avulla. Voidaan generoida n värin gradientti ja käyttää näitä värejä kuvassa. Voidaan pyytää käyttäjää liikuttamaan kuvan objekteja ja ratkaisun ollessa oikea näyttää palauteteksti. Javascriptin avulla voidaan myös tuottaa ääniä – siispä voidaan säätää siniaaltojen taajuuksia, yhdistää kolme siniaaltoa soinnuksi, piirtää soinnun kuvaaja ja kuunnella vastaava ääni.

Millaisia matemaattisia havainnollistuksia JSXGraphilla voi tehdä? Mihin havainnollistuksiin JSXGraph luonteeltaan soveltuu ja mihin selaimen suorituskyky riittää?

JSXGraphilla voi ratkaista dynaamisesti 2D-ongelmia, kuten määrittää painopiste viisikulmiolle. Jos on annettu n tason pistettä eli tumaa, voidaan piirtää tason Voronoi-solut: tumaa ympäröivä solu koostuu niistä tason pisteistä, joille kyseinen tuma on lähimpänä.

Pienellä vaivalla JSXGraphin avulla voi havainnollistaa myös kolmiulotteisia kappaleita. Voidaan tehdä havainnollistuksia monitahokkaista, maapallosta tai pinnoista, kuten kahden muuttujan funktion kuvaajasta.

Voidaan tarkastella reaalimaailman ilmiöitä. Voidaan esimerkiksi vertailla karttaa ja samalta alueelta otettua ilmakuvaa, jolloin tarvitaan perspektiivikuvauksia. Onneksi valmiita perspektiivikuvauksien koodeja löytyy internetin keskustelupalstoilta.

JSXGraph auttaa myös vähemmän matemaattisten havainnollistusten tekemisessä. Tällöin kuvan koodia voi generoida muillakin ohjelmointikielellä, kuten PHP:llä, joka voi poimia ja tarjoilla JSXGraphille tiedostoja verkkopalvelimelta tai taulukkomuodossa olevia tietoja.

Joustavan matemaattisen päättelyn automaattinen tarkastaminen lineaaristen (itseisarvo)(epä)yhtälö(ryhmien) tapauksessa

Antti Valmari
Jyväskylän yliopisto, Informaatioteknologian tiedekunta

Lineaarisen yhtälöryhmän voi tyypillisesti ratkaista monella tavalla, ja jokin tapa voi olla huomattavasti vähätöisempi kuin jokin toinen. Tämä korostuu kun siirrytään epäyhtälöihin ja mukaan tulevat itseisarvot. Sellaisilla tehtävillä voi auttaa opiskelijoita nousemaan mekaanisen ratkaisemisen yläpuolelle, miettimään ratkaisustrategioita ja etsimään mahdollisia oikoteitä.

Olemme rakentaneet matemaattista logiikkaa syvällisesti hyödyntävän ohjelman, joka tarkastaa opiskelijan esittämän ratkaisun vaihe vaiheelta riippumatta siitä, mitä reittiä se etenee. Ratkaisu muodostuu symbolien $\Rightarrow$, $\Leftrightarrow$, $\Leftarrow$ ja $\equiv$ avulla esitetystä päättelystä, jonka sisällä voi olla alipäättelyitä. (Ali)päättelyllä voi olla lähtöoletus, esimerkiksi $x > y + 1$. Opiskelija voi jakaa tehtävän tapauksiin, ratkaista ne yksi kerrallaan alipäättelyinä ja yhdistää tulokset.

Esimerkkinä ohjelman kyvyistä näytämme, miten sille voi esittää ja miten se tarkastaa oikotietä hyödyntävän ratkaisun George Pólyan kuuluisassa kirjassa ”Ratkaisemisen taito” älypähkinänä esitetylle neljän yhtälön ryhmälle:

$$\begin{array}{rcr} x + 7y + 3v + 5u &=& 16 \\ 8x + 4y + 6v + 2u &=& -16 \end{array} \qquad \begin{array}{rcr} 2x + 6y + 4v + 8u &=& 16 \\ 5x + 3y + 7v + u &=& -16 \end{array}$$

(Epä)yhtälössä saa olla myös murtofunktioita, jos se palautuu lineaariseksi kertomalla nimittäjät pois. Tämä tarjoaa mahdollisuuden harjoituttaa nimittäjien nollakohtien oikeaa käsittelyä. Kuten oheinen kuva näyttää, ohjelma ei hyväksy päättelyaskelta $\frac{|2x-9|-3}{|x+1|-4} = 0 \Leftrightarrow |2x-9|-3 = 0$, koska kun $x = 3$, on vasemmalla puolella nollalla jako mutta oikea puoli on tosi. Ohjelma hyväksyy $\frac{|2x-9|-3}{|x+1|-4} = 0 \Leftrightarrow |2x-9|-3 = 0 \wedge |x+1|-4 \neq 0$, koska siinä nimittäjän nollakohdat on rajattu oikealla puolella pois.

$$\frac{|2x-9|-3}{|x+1|-4} = 0 \;\Leftrightarrow\; |2x-9|-3 = 0$$

Relation does not hold when $x = 3$

left = U
right = T

Ohjelman voi tapauskohtaisesti säätää vaatimaan, että lopputulos on sievennetty tiettyyn muotoon. Ohjelma saadaan muun muassa ilmoittamaan, että $-1 < x \leq 3 \vee 2 < x < 4$ on matemaattisesti oikein, mutta yksinkertaisempikin oikea vastaus on olemassa.

Koska ohjelman toiminta perustuu päättelyn pätevyyden ja lopputuloksen muodon tarkastamiseen eikä mallivastaukseen vertaamiseen, sitä voi käyttää ilman että opettaja syöttää sille mallivastauksia tms. Toisaalta ohjelman avulla voi teettää myös tehtäviä, joissa kysymys esitetään opiskelijalle sanallisesti, ja opiskelijan on itse muotoiltava se (itseisarvo)(epä)yhtälö(ryhmäksi). Tällöin opettajan on syötettävä ohjelmalle mallivastaus tms.

Ohjelman lähdekoodi on vapaasti saatavana GPL3-lisenssin alaisena, joten kuka tahansa voi asentaa ohjelman omaan laiteympäristöönsä. Vaihtoehtoisesti käyttäjä voi pyytää lupaa käyttää esimerkiksi Jyväskylän yliopiston palvelimelle asennettua versiota.

Esitys perustuu CSEDU 2021 -konferenssissa julkaistuun paperiin, jonka voi lukea täältä: `https://jyx.jyu.fi/handle/123456789/75712`. Sen esimerkit voi itse (ilman mitään kirjautumista tms.) kokeilla täältä: `http://users.jyu.fi/~ava/CSEDU21_examples.html`.

Digital twins of forest ecosystems

Tuomo Kauranne, Arbonaut Ltd.

Forests are increasingly important for our coping with the twin hazards of climate change and loss of biodiversity. Understanding forest ecosystems is greatly enhanced by digital twins of forest ecosystems that represent each forest as a web of dependencies between its physical and biological component parts. In addition to the forest and trees in it, these parts comprise circulation of water in the ecosystem, soil properties, biodiversity hotspots and also an assessment of ecological risks such as wildfires or bark beetle infestations. In this talk we survey a range of remote sensing modalities and mathematical modelling approaches relevant to forest ecosystems inventory and assessment of ecological risks.

Sawing Optimization, a Tandem Forest values project

Sebastian Springer, University of Oulu

The main steps of the sawing optimization process we consider in this project consist in getting accurate CT images from limited sequentially obtained projection data, segmenting the reconstructed trunk, and finally optimizing the cutting parameters.

The goal of all the stakeholders involved in the sawing processes is to produce the highest possible quality boards from each log, while minimizing the waste of time and resources. Among the most important factors determining the quality of a panel, and consequently also its value, one can certainly find the resistance to bending and the absence of foreign objects like metallic nails. It is therefore important to avoid the presence of wood knots on the lateral edge of the boards as it can considerably compromise the resistance to bending of this latter.

In this talk we will present the imaging pipeline and various approaches to obtain a reconstruction of the trunks from classic variational approaches to learned methods. We then compare precision of the reconstructions in terms of precision of the subsequently obtained segmentation with respect to an accurate reconstruction from 360 angles.

Deep Learning Based OFDM Physical-Layer Receiver for Extreme Mobility

Jaakko Pihlajasalo
Tampere University

In this talk, we present a machine learning (ML) based physical layer receiver solution for demodulating OFDM signals that are subject to a high level of nonlinear distortion. In addition, we present some mathematical background of the OFDM signals and the utilized neural network methods. This talk is based on a paper presented in PIMRC 2021 conference, adjusted to be more suitable to an audience with mathematical background.

Brief introduction on the structure and signal processing of OFDM signals is presented in addition to a description of the effect of power amplifier on the signals. Furthermore, some basic concepts of neural networks such as optimization algorithms, convolutional neural networks and residual networks are presented.

Specifically, a novel deep learning based convolutional neural network receiver is devised, containing layers in both time- and frequency domains, allowing to demodulate and decode the transmitted bits reliably despite the high error vector magnitude (EVM) in the transmit signal. Extensive set of numerical results is provided, in the context of 5G NR uplink incorporating also measured terminal power amplifier characteristics.

The obtained results show that the proposed receiver system is able to clearly outperform classical linear receivers as well as existing ML receiver approaches, especially when the EVM is high in comparison with modulation order. The proposed ML receiver can thus facilitate pushing the terminal power amplifier (PA) systems deeper into saturation, and thereon improve the terminal power-efficiency, radiated power and network coverage.

Intermediate logics in the team semantics setting

Fan Yang
University of Helsinki

In this work, we explore intermediate logics in the team semantics setting. Team semantics was introduced by Hodges [3], and later advanced by Väänänen in dependence logic [6], and adopted independently in inquisitive logic [2]. Both dependence and inquisitive logic were introduced as extensions of classical logic. Recently several authors have defined different intuitionistic logic-based dependence/inquisitive logic [5, 1, 4]. Our starting point is [1]. The key idea of (intuitionistic) propositional team semantics is that formulas are evaluated in (intuitionistic) Kripke models $\mathfrak{M} = (W, R, V)$ with respect to *sets* $t \subseteq W$ of possible worlds (called *teams*). We also extend the language $[\bot, \wedge, \vee, \to]$ of intuitionistic logic (IPC) with a disjunction $\mathbin{\mid\!\vee}$ on the team level. The system tIPC of the logic in [1] consists of IPC axioms for $[\bot, \wedge, \mathbin{\mid\!\vee}, \to]$, some simple axioms for $\vee$, and the Split axiom $\alpha \to (\phi \mathbin{\mid\!\vee} \psi) \to (\alpha \to \phi) \mathbin{\mid\!\vee} (\alpha \to \psi)$ with $\alpha \in [\bot, \wedge, \vee, \to]$.

We provide two alternative approaches to define intermediate team-based logics, by modifying tIPC with axioms with $\vee$ or $\mathbin{\mid\!\vee}$. Given an intermediate logic $\mathsf{L} = \mathsf{IPC} \oplus \Delta$ with Δ a set of $[\bot, \wedge, \vee, \to]$-axioms, the first approach defines an intermediate logic tL by closing the set $\mathsf{tIPC} \cup \{\alpha(\vec{\beta}/\vec{p}) \mid \alpha \in \Delta,\ \beta_i \in [\bot, \wedge, \vee, \to]\}$ under Modus Ponens. We show that if L is complete for a class F of frames, then tL is also complete for F, provided that L has disjunction property or is canonical. In the second approach we replace the Split axiom of tIPC with other $[\bot, \wedge, \mathbin{\mid\!\vee}, \to]$-axioms. This amounts to changing the underlying structure of teams from $(\wp(W), \supseteq)$ to $(\wp(W), \succcurlyeq)$ with an arbitrary partial order $\succcurlyeq$, and thus generalizing the standard team semantics.

This is joint work with Nick Bezhanishvili.

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On Cherlin-Zilber Algebraicity conjecture

Ulla Karhumäki, University of Helsinki

After Morley proved his celebrated Categoricity theorem, a certain notion of dimension—today called the *Morley rank*—was recognised fundamental in the studies of model theory. This notion of dimension generalises geometers *Zariski dimension*, that is, the class of groups of finite Morley rank generalises the class of algebraic groups over algebraically closed fields. While the former class is strictly broader than the latter class (being for example closed under the formation of finite products), these two classes are closely connected. This connection was first highlighted by the famous Cherlin-Zilber Algebraicity conjecture proposed in the 1970s independently by Cherlin and Zilber:

Conjecture 1 *Infinite simple groups of finite Morley rank are isomorphic to algebraic groups over algebraically closed fields.*

Though the Cherlin-Zilber conjecture remains widely open, strong results (dealing some parts of the conjecture) are proven through the connection with (techniques of) finite group theory. In my talk, I will first survey the greatest achievements of the topic so that the current state of the Cherlin-Zilber conjecture may be explained. Then, I will present some results together with future research goals related to an ongoing research program in which we approach the Cherlin-Zilber Conjecture via 'tight configurations' of finite Morley rank.

Independence Logic and Quantum Physics

Joni Puljujärvi

Quantum physics provides a source of many interesting dependence and independence concepts. In a join work with Samson Abramsky and Jouko Väänänen, we study some of these concepts, especially those related to hidden variables and non-locality, through the lens of team semantics and probabilistic team semantics. Many of the same results were independently acquired by Albert & Grädel and studied in their recent paper.

Team semantics is a semantic framework for studying logics of dependence and independence, introduced by Hodges in 1997 and popularized by Väänänen in 2007. Independence logic is a team-based logic that can express independence properties of teams. Probabilistic independence logic, introduced in 2018 by Durand et al., is a probabilistic variant of independence logic, concerning a probabilistic generalization of teams.

On a paper in 2013, Abramsky introduced a relational framework for studying dependence and independence properties of hidden-variable models. This framework can be very conveniently translated to team semantics, making dependence and independence properties of relational hidden-variable models into formulas of independence logic and relationships between the properties instances of logical consequence between the formulas. Many of the consequences we are able prove using axioms found in previous work on independence logic. The famous no-go theorems of quantum mechanics become instances of failure of logical consequence, demonstrated by explicit counter-example team constructions.

A similar team-semantical treatment can be given to a probabilistic framework of Brandenburger & Yanofsky from 2008 (on which Abramsky's relational framework is also based), translating probabilistic independence notions into formulas of probabilistic independence logic. The probabilistic properties of hidden-variable models become simply probabilistic variants of the corresponding formulas from the relational case. Probabilistic independence logic also happens to satisfy all the axioms we use in the relational setting, so we obtain all the results that are provable in ordinary independence logic also for probabilistic independence logic.

The study of independence notions in quantum physics also leads to new discoveries in both ordinary and probabilistic independence logic. Such include the concept of probabilistic realizability of a team and a form of quantum realizability of a team, which turns out to be an undecidable (but recursively enumerable) property of teams.

Parallel sessions B

8 Number theory with a modern twist (chair: A-M Ernvall-Hytönen)

Niklas Miller: *Lattices from algebraic number fields*
Njah Rahinatou: *RLWE/PLWE for cyclotomic number fields*
Jorma Merikoski: *Pienimmän positiivisen primitiivijuuren yläraja*

9 Stochastics (chair: Jonas Tölle)

Lauri Viitasaari: *On existence and uniqueness of the solution for stochastic PDE's*
Gerardo Barrera: *Quantitative control of Wasserstein distance between Brownian motion and the Kac telegraph process*
Tommi Sottinen: *A New characterization of Brownian motion as isotropic i.i.d.-component Lévy process*

10 Functional Analysis 2 (chair: Hans-Olav Tylli)

David Norrbo: *Spectral properties of multipliers on Bergman–Sobolev spaces and Bloch-type spaces*
Fanglei Wu: *Integral operators mapping into H^∞*
Henrik Wirzenius: *Closed ideals in the algebra of compact-by-approximable operators*

11 Applied Mathematics in Africa (chair: Lassi Roininen)

Denis Ndanguza: *Modeling of Deep Methane Gas Extraction from Lake Kivu*
Yusuf Yusuf: *Modelling Extreme Events: The Frequentist and Bayesian Perspectives*
Abdu Mohammed Seid: *The Spatiotemporal Dynamics of Water Hyacinth over Lake Tana of Ethiopia using Hidden Potts prior and Dynamic Linear Models*

12 Oppijalähtöinen matematiikan opetus (chair: Johanna Rämö)

Juho Tiainen: *Työelämäsisällöt matematiikan opetuksessa*
Marika Toivola ja Mari Kaunisto: *Käänteinen arviointi insinöörimatematiikassa*
Jokke Häsä: *Integrating generic skills in student-centred mathematics teaching*

13 Uncertainty quantification (chair: Vesa Kaarnioja)

Harri Hakula: *On Solution of Elliptic Partial Differential Problems on Random Domains*
Philipp Guth: *Surrogates in PDE-Constrained One-Shot Optimization under Uncertainty*
Vesa Kaarnioja: *High-dimensional kernel interpolation over lattice point sets with application to uncertainty quantification*

14 Mathematical logic 2 (chair: Antti Kuusisto)

Kerkko Luosto: *Dimensions of families of sets with applications to team semantics*
Daniele Mundici: *Handling boolean events consistently*
Reijo Jaakkola: *Ordered fragments of first-order logic*

27 Ekonometria (chair: Henri Nyberg)

Leena Kalliovirta: *Demand and Supply Elasticities in the Finnish Roundwood Markets in 1995-2017*
Juho Koistinen: *Estimation of impulse-response functions with dynamic factor models: a new parameterization*
Visa Kuntze: *Similarity-augmented structural vector autoregression: The effects of forward guidance shocks in different monetary policy conditions*

Lattices from algebraic number fields

Niklas Miller

In this talk, I will discuss lattices which are constructed from algebraic number fields. Number fields can be seen as geometric objects by embedding them into $\mathbb{C}^n$ via the Minkowski embedding. Under this embedding, the image of the ring of integers of a number field, and its ideals, form lattices. The properties of such lattices, e.g. volume, packing density and minimum product distance, are related to properties of the underlying number field, such as its discriminant.

Algebraic lattices have applications in coding theory, especially in wireless communications. For instance, in a wireless communication model called Rayleigh fading channel, a desirable property of the lattice code is a high minimum product distance. For unimodular algebraic lattices coming from principal ideals of ring of integers, the minimum product distance is determined by the inverse of the square root of the discriminant of the underlying number field, which means that the complex problem of finding lattices with high minimum product distance can be reduced to the problem of finding number fields with small discriminant.

The problem of finding the densest packing of spheres with equal radii in n-dimensional Euclidean space is a problem with a long history. In the 17th century, Kepler conjectured that the face-centered cubic lattice packing is the densest one in dimension 3, but this was not proved until the late 1990's. The densest lattice packings, with spheres centered at lattice points, are only known in dimensions 1–8 and 24. We will take a look at how some of the densest lattice packings can be constructed from algebraic number fields.

RLWE/PLWE for cyclotomic number fields

Rahinatou Njah

RSA, is threatened by the rapid progress in quantum information, as such a need to protect our systems from post-quantum attacks, which leads us to lattice based cryptography(LBC). Ring learning with error problem, is one of the promising technology of LBC.

We study the equivalence between the ring learning with errors and polynomial learning with errors problems for cyclotomic number felds, via polynomial noise increase. We give sharper bounds in the case where the conductor is divisible by at most six primes.

Pienimmän positiivisen primitiivijuuren yläraja

Jorma Merikoski
Tampereen yliopisto

Olkoon n positiivinen kokonaisluku ja p pariton alkuluku. Seppo Mustosen [1, 2] kokeiden mukaan kongruenssin

$$x^n + y^n \equiv 0 \pmod{p}$$

ratkaisupisteet ovat tasavälisillä suorilla, joiden suuntien lukumäärä on $\mathrm{syt}(p-1, n)$. Kaikki näiden suorien kokonaislukupisteet ovat ratkaisuja, ja kukin epätriviaali ratkaisu on täsmälleen yhdellä tällaisella suoralla. Päätin yrittää ratkaista tätä kongruenssia, jolloin voisin todistaa, että Mustosen koetulokset ovat yleisesti voimassa. Pikaisella kirjallisuuden jäljittämisellä en löytänyt ratkaisua.

Jouduin siinä työssä perehtymään primitiivijuuriin modulo p. Minusta oli kiinnostavaa, että pienin positiivinen primitiivijuuri $g(p)$ näyttää p:hen verrattuna pysyvän erittäin pienenä. Niinpä aloin etsiä kirjallisuudesta ylärajoja $g(p)$:lle. Minulle oli melkoinen yllätys, ettei edes (todella huonoa) ylärajaa $g(p) < p$ ole todistettu yleisesti päteväksi. (Triviaalisti $g(p) \leq p$.)

Parempia (mutta edelleen huonoja) ylärajoja löytyy tekemällä sopivia p:tä koskevia oletuksia. Esimerkiksi [3]

$$g(p) \leq \sqrt{p} - 2,$$

kun $409 < p < 2.5 \cdot 10^{15}$ tai $p > 3.38 \cdot 10^{71}$. Välissä oleva ”harmaa alue” jää otaksuman varaan. Siitä päästään eroon luottamalla yleistettyyn Riemannin hypoteesiin, jolloin saadaan seuraava vahvempi tulos [4]. Olkoon $\hat{g}(p)$ pienin sellainen alkuluku, joka on primitiivijuuri modulo p. Tällöin

$$\hat{g}(p) < \sqrt{p} - 2,$$

kun $p > 2791$.

Timo Tossavainen (Luulajan teknillinen yliopisto) on etsinyt kokeellisesti $g(p)$:n ylärajoja, kun $p < 10^9$. Tulosten perusteella saadaan otaksumia sellaisista rajoista, jota pätevät kaikille p:lle.

Viitteet

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On existence and uniqueness of the solution for stochastic partial differential equations

Lauri Viitasaari
Uppsala University

In this talk we consider existence and uniqueness of the solutions to a large class of stochastic partial differential equations of form $\partial_t u = L_x u + b(t,u) + \sigma(t,u)\dot{W}$, driven by a Gaussian noise $\dot{W}$, white in time and spatial correlations given by a generic covariance γ. We provide natural conditions under which classical Picard iteration procedure provides a unique solution. We illustrate the applicability of our general result by providing several interesting particular choices for the operator L_x under which our existence and uniqueness result hold. In particular, we show that Dalang condition given in Dalang (1999) is sufficient in the case of many parabolic and hypoelliptic operators L_x.

The talk is based on a joint work with B. Avelin (Uppsala University).

Quantitative control of Wasserstein distance between Brownian motion and the Kac telegraph process

Gerardo Barrera & Jani Lukkarinen
University of Helsinki, Department of Mathematics and Statistics

The so-called Goldstein-Kac telegraph process is an example of a random evolution with simple structure. It describes the movement of a particle which starts at time zero from the origin and moves with a finite constant speed on the line. The initial direction of the motion, positive or negative, is chosen at random with the same probabilities. The changes of direction are driven by a homogeneous Poisson process of a positive constant intensity. In other words, when a jump occurs in the Poisson process, the particle instantaneously takes the opposite direction and keeps moving with the same speed (it just flips the sign of the velocity) until the next jump in the Poisson process happens, then it takes the opposite direction again, and so on.

In this talk, I present a non-asymptotic process level control between the telegraph process and the Brownian motion with suitable diffusivity constant via a Wasserstein distance with quadratic average cost. The proof relies on coupling techniques such as the coin-flip coupling and the Komlós-Major-Tusnády coupling.

A New characterization of Brownian motion as isotropic i.i.d.-component Lévy process

Tommi Sottinen
University of Vaasa

The Brownian motion is arguably the most important stochastic process there is. It has a long history in particle physics dating back to at least to the Roman poet and philosopher Lucretius and his scientific poem *De rerum natura* ca. 50 BC. Since then, the Brownian motion has proven to be central also in such various fields as physics, economics, quantitative finance, and evolutionary biology, just to mention few.

The Brownian motion has many characterizations. It is, for example, the scaling limit of random walks, the independent-increment stationary-increment Gaussian process, the 1/2-self-similar stationary-increment Gaussian process, the Markov process with Laplacian as its generator, the continuous Lévy process, the continuous local martingale with identity function as its bracket.

We provide a new characterization of the Brownian motion as the isotropic Lévy process with independent identically distributed components. Our proof will be short and simple, and highly non-elementary.

The talk is based on the article [1].

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Spectral properties of multipliers on Bergman-Sobolev spaces and Bloch-type spaces

David Norrbo

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$. We characterize the spectra of the opertors M_u associated with the pointwise multipliers u acting on Banach spaces of analytic functions on $\mathbb{D}$ satisfying some general conditions. These spaces include Bergman-Sobolev spaces $A^p_{\alpha,\beta}$ and Bloch-type spaces $\mathcal{B}_\alpha$. Moreover, we describe the essential spectra of M_u in most of the aforementioned spaces, in particular, in those spaces for which the set of multipliers is a subset of the ball algebra.

This talk is based on joint work with Mikael Lindström and Santeri Miihkinen.

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Integral operators mapping into H^∞

Fanglei Wu

In this talk, we will present several neat and useful conditions in terms of Maclaurin coefficients of the symbol g that can be used to test if the integral operator $T_g(f)(z) = \int_0^z f(\zeta)g'(\zeta)d\zeta$ is either bounded or compact from a space X of analytic functions in the unit disc to H^∞. The choices of X that will be considered contain the Hardy and the Hardy-Littlewood spaces, the Dirichlet-type spaces D^p_{p-1}, as well as the classical Bloch space and BMOA. This is a joint work with Prof. J. Peláez and Prof. J. Rättyä.

Closed ideals in the algebra of compact-by-approximable operators

Henrik Wirzenius
University of Helsinki

I will discuss joint work with Hans-Olav Tylli on the closed ideals of the compact-by-approximable algebra $\mathfrak{A}_X := \mathcal{K}(X)/\mathcal{A}(X)$ on Banach spaces X failing the approximation property. Here $\mathcal{K}(X)$ denotes the compact operators $X \to X$ and $\mathcal{A}(X) = \overline{\mathcal{F}(X)}$ is the uniform closure of the bounded finite rank operators $X \to X$.

The results include the following: (i) if X has cotype 2, Y has type 2, $\mathfrak{A}_X \neq \{0\}$ and $\mathfrak{A}_Y \neq \{0\}$, then $\mathfrak{A}_{X\oplus Y}$ has at least two closed ideals; (ii) there are closed subspaces $X \subset \ell^p$ for $4 < p < \infty$ and $X \subset c_0$, such that $\mathfrak{A}_X$ has a non-trivial closed ideal.

Some of the results involve the Banach operator ideal $\mathcal{QN}_p$ of the quasi p-nuclear operators for $p > 4$ and, time allowing, I will briefly present a recent result on $\mathcal{QN}_p$ which yields that there is a closed subspace $X \subset c_0$ such that $\mathfrak{A}_X$ has two infinite decreasing chains of closed ideals.

The talk is based on the paper: H.-O. Tylli, H. Wirzenius, *Closed ideals in the algebra of compact-by-approximable operators*, Journal of Functional Analysis, Volume 282, Issue 4, 2022.

Modeling of deep methane gas extraction from Lake Kivu

Denis Ndanguza
University of Rwanda

Lake Kivu, located on the border of Rwanda and the Democratic Republic of Congo is one of the world's unique and most dangerous lakes. The uniqueness and the danger of Lake Kivu arise from the large amount of carbon dioxide (CO_2) and methane (CH_4) gases dissolved in the deep waters of the lake. Most of the methane is stored in the deep part of the lake, indicated as the Resource Zone in the length of 500 m. These gases are a natural hazard, as they could potentially lead to a gas eruption from the lake if their concentrations increase further. The best approach to eliminate any risk for a gas eruption would be to completely remove all the gases from Lake Kivu immediately. Currently, the dissolved CH_4 is being extracted from lake and used for power production. However, the gas can not be fully exterminated from the lake due to a metabolism generating the gas and its future development is not well predicted [2]. Previous researchers [1] have modeled the gas extraction from lake Kivu by considering the inflow rate as constant. The aim of this paper is to consider the inflow as a variable function generating the methane gas by chemical reactions inside the lake. Taking into consideration the inflow rate as a variable, the concentration profiles through the Lake will be computed and interpreted accordingly.

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Modelling Extreme Events: The Frequentist and Bayesian Perspectives

Yusuf Oluwatoki Yusuf
Tampere University

Extreme events are processes associated with occasional deviation from the usual state of a system. These events are observed in different field of engineering, natural and social sciences. Examples of extreme events include financial crises, rogue waves, drought, flood, earthquakes, faults in industrial processes, and many others in several application areas of dynamical systems. Clearly, some systems are either space and/or time dependent hence the extreme events of these systems are considered differently. An application area is the dynamics and mechanism of ocean waves which is a spatial dynamical system [2]. Similarly, extreme climate events are yet another application which posses severe impact of climate change on human lives, economy, infrastructure and the environment in general. With several observations, statistical methods of estimation can be used to recover the characterising elements of the system which are now called parameters of the model. The frequentist and Bayesian methods play prominent roles in estimating the parameters of this statistical models. The frequentist method uses conditional distributions of data given specific hypotheses. The Bayesian approach uses Bayes theorem to combine the observational data with subjective beliefs called priors [1]. The two methods provide grounds for statistical inferences and decision making.

The use of statistical tools such as extreme value theory [3] help to understand what is expected in terms of the intensity of extremes over the coming years. Bayesian approaches for estimating parameters of systems and phenomena that appear to vary in a random manner such as Markov Chain Monte Carlo (MCMC) [4] can be used to recover the parameters of such systems in the presence of noisy observations. I will present flood in Axim region of Ghana as a case study on extreme event for the frequentist approach, and ocean waves in the North Sea as a case study on extreme event for the Bayesian approach at the Finnish Mathematics days 2022 for the Applied Mathematics in Africa session.

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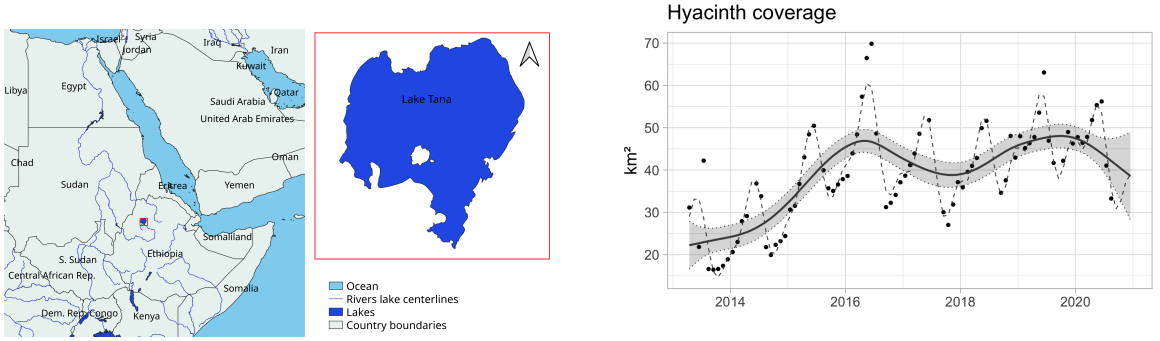
The Spatiotemporal Dynamics of Water Hyacinth over Lake Tana of Ethiopia using Hidden Potts prior and Dynamic Linear Models

Abdu Mohammed Seid, Department of Mathematics, Bahir Dar University, Ethiopia.
Matthew Moores, NIASRA, University of Wollongong, Australia.
Marko Laine, Finnish Meteorological Institute, Helsinki, Finland.
Lassi Roininen, Lappeenranta-Lahti University of Technology, Finland.

The invasion of water hyacinth over lake Tana of Ethiopia, one of the largest lake in Africa and the source of the Blue Nile, has caused significant problems of biodiversity and resulting in high attentions and costs for eradicating this invasive aquatic weed. The objective of this study is therefore to investigate the spatiotemporal dynamics of the hyacinth over the lake and to identify the status of hyacinth infestation by classifying remotely sensed imageries of the lake. A time-series of multisensor high resolution imagery obtained from Landsat-8/OLI during the period 2013–2020 are analyzed to study the temporal, seasonal and spatial variability of the invasion that started in 2011 in lake Tana. We used a Bayesian model based image classification with Hidden Potts prior assumption to investigate the spatiotemparal distribution of the hyacinth over the lake during the study period. We also analyzed its seasonal and trend dynamics using a Dynamic Linear Model.



(a) Study Area: Lake Tana, Ethiopia.

(b) Seasonality and Trend Analysis of hyacinth coverage over Lake Tana using DLM.

Figure 1: Study area and Time series Analysis result.

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Työelämäsisällöt matematiikan opetuksessa

Juho Tiainen

”Ope, mihin tätä tarvitaan?” oppilaan suusta oppitunnilla tuleva kysymys lienee monelle matematiikkaa peruskoulussa tai toisella asteella opettaneelle tuttu. Oman kokemukseni perusteella vastausta oppilaan kysymykseen saattaa joutua hetken miettimään, ja samalla opettajan aivot rekisteröivät havainnon, että muista oppilaista osa pudottaa kynän kädestään. Matematiikan tarpeellisuus on jälleen kyseenalaistettu.

Tällaiset kokemukset matematiikan opettamisesta ovat jääneet mietityttämään minua siinä määrin, että aloin vuonna 2019 suunnitella toimenpiteisiin ryhtymistä matematiikan tarpeelliseksi kokemisen tason nostamiseksi. Olisi tietenkin lapsenmielistä uskoa, että tämänkaltainen ongelma saataisiin koskaan yksiselitteisesti ratkaistua, joten tavoitteeni on tarjota opettajille eväitä työelämäsisältöisen opetusmateriaalin kehittämiseen. Projektissa ovat mukana niin yliopiston henkilökunta, opiskelijat kuin peruskoulun matematiikan opettajat ja oppilaat.

Viitekehyksenä käytetään Eccles'n Wigfieldin odotusarvoteoriaa (expectancy-value theory), jossa hyödyllisyyden kokemus (utility value) nähdään eräänä tekijänä, jolla on positiivinen yhteys esimerkiksi valintoihin (choices), saavutuksiin ja suoriutumiseen (achievement, performance) sekä muun muassa vaivannäköön, sitoutumiseen ja sinnikkyyteen (effort, engagement, persistence). Työelämäsisältöinen opetusmateriaali määritellään tarinaksi, jossa autenttiset työtehtävät esitetään matematiikan harjoitustehtävinä. Tarinan ohessa voi myös olla tietoa liittyen urapolkuihin ja koulutusmahdollisuuksiin. Näin ollen työelämän mahdollisuuksilla ja pyritään luomaan oppilaalle kokemuksia matematiikan tarpeellisuudesta, minkä lisäksi opinto-ohjaus integroituu luontevasti oppiaineeseen

Opetusmateriaalin valmistamiseen osallistuivat Itä-Suomen yliopiston Joensuun kampuksen matematiikan aineenopettajaopiskelijat syksyllä 2021 järjestetyllä opintojaksolla. Opetusmateriaali kootaan yhteen tai useampaan verkkoympäristöön, ja interventiossa yläkoulun oppilaat tutustuvat materiaaleihin ja ratkaisevat tarinoiden sisältämiä matematiikan tehtäviä. Tavoitteina on selvittää, millaisia vaikutuksia interventiolla saavutetaan matematiikan opiskeluun vielä yläkoulun aikana sekä miten interventio vaikuttaa oppilaan uraintentioihin ja koulutusvalintoihin. Kvantitatiiviset mittaukset ja oppilaiden tekemät ammattinimikelistaukset toteutetaan ennen ja jälkeen intervention sekä viivästetysti muutaman kuukauden päästä interventiosta. Lisäksi myöhemmin seurataan interventioon osallistuneiden oppilaiden toteutuneita koulutusvalintoja, kuten esimerkiksi sitä, onko lukiossa opiskeleva oppilas valinnut pitkän matematiikan.

Lopputuotteena on tavoitteena tuottaa opetusmateriaalia, jota on kehitetty yläkoulun oppilailta ja opettajilta saadun palautteen perusteella. Tällä hetkellä ensimmäiset opetusmateriaalit ovat valmistuneet, mutta itse tutkimustyössä ollaan vasta materiaalin esitestausvaiheessa, jossa kerätään yläkoulun oppilailta kommentteja eri tavoin toteutetuista tarinoista, minkä lisäksi halutaan tietoa siitä, miksi oppilas kiinnittyy joihinkin tarinoihin ja päättää ratkaista näiden tarinoiden sisältämiä matematiikan tehtäviä. Lisäksi tulevien vuosien aikana on tavoitteena kehittää aineenopettajaopiskelijoille suunnattavaa opintojaksoa oppijalähtöisempään suuntaan siten, että opetusmateriaalissa esiintyvät ammatit olisivat yläkoulun oppilaiden toiveammatteja.

Käänteinen arviointi insinöörimatematiikassa

Marika Toivola ja Mari Kaunisto

Tarkastelemme tässä esityksessä käänteistä matematiikan arviointia Satakunnan ammattikorkeakoulun insinöörimatematiikan opetuksessa, jossa toteutamme käänteistä oppimista yhteisopettajuutena.

Käänteisen arvioinnin käsite on alkuaan luotu tukemaan käänteisen oppimiskulttuurin rakentumista. Sen tarkoitus on yhtäältä ohjata oppilaita toimimaan asiantuntijan tavoin omassa oppimisessaan ja toisaalta auttaa opettajaa sellaisen oppimiskulttuurin rakentamisessa, joka tukee opiskelijan itse- ja yhteisohjautuvuuden kehittymistä. Itseohjautuvuus edellyttää itsearviointitaitoja, joiden kehittymistä edistetään itse- ja vertaisarvioinnilla. Vertaisarviointi liittyy ajatukseen toisten oppimisen tukemisesta sekä kyvystä hyödyntää muita omassa oppimisessaan. Itsearviointi liittyy ajatukseen oman oppimisen omistajuudesta. Käänteisessä arvioinnissa itse- ja vertaisarvioinnin keskiössä ei siis ole oman tai vertaisen tuotoksen arvosteleminen. Vaikka arviointi sisältää osaamisen mittaamista, mittaamisesta itsestään ei missään vaiheessa tule tulostavoite. Itse- ja yhteisohjautuvuuden tukemisen lisäksi kiinnitimme huomiota siihen, minkälaista matemaattista osaamista insinööriltä vaaditaan. Työelämä peräänkuuluttaa insinöörejä, joilla on uskallusta kokeilla uutta, uskallusta epäonnistua sekä kykyä tehdä yhteistyötä.

Käytännössä arviointimme on suunniteltu siten, että se pakottaa opiskelijat epäonnistumaan ja lisää ymmärrystä epäonnistumisen merkityksestä oppimiskulttuuri rakentumiselle. Algebran kurssillamme oli kolme formatiivista arviointisykliä, joista jokainen oli kaksivaiheinen ja perusperiaatteeltaan samanlainen. Sama koe tehtiin kahdesti. Ensimmäinen vaihe oli ns. yksilösuoriutumisen vaihe, jossa opiskelija nousi epäonnistumisen tasolleen. Käytännössä se tarkoitti kolmesta eritasoisesta kokeesta (tasot 1-2, 3-4 ja 5) sen valitsemista ja tekemistä, jonka tehtävistä arvioi saavansa kolmasosan oikein. Toinen vaihe oli yhteissuoriutumisen vaihe, jossa opiskelijat hyödynsivät kurssitovereitaan kokeen loppuunsaattamisessa. Lähtökohtaisesti tasoilla 1-2 ja 3-4 paremman arvosanan sai, jos yhteissuoriutumisen jälkeen tehtävistä 70 % oli oikein. Arvosanan 5 sai, kun sen tehtävistä 30 % oli oikein. Arvosanatavoitetta 5 vastaavista tehtävistä yksikään ei ollut sellainen, josta selviytyisi aiempia toimintatapoja toistamalla. Tällä tasolla vaadimme kyvykkyyttä soveltaa oppimansa uudessa kontekstissa.

Esityksessä näytämme kokeiden tulosjakaumat ja jaamme kokemuksiamme siitä, kuinka arviointisyklejä kehitettiin yhteistyössä 126:n opiskelijan kanssa. Arviointi nähtiin yhteisöllisenä prosessina, ei menettelytapana, jonka mekaaninen soveltaminen sellaisenaan tekisi arvioinnista formatiivista. Käänteisen arviointi kannusti meitä opettajina toimimaan tavalla, joka toi tilaa opiskelijoiden oman äänen kuulumiselle ja auttoi näkemään arvioinnin ja sen tuomat mahdollisuudet uudella tavalla.

Integrating generic skills in student-centred mathematics teaching

Jokke Häsä

Generic skills, such as communication and teamwork skills, are considered important in learning and for future employment. There is evidence in literature that the most effective way to teach these skills is to integrate them into content courses, instead of offering separate courses. However, these skills are rarely taught explicitly in mathematics courses. This may be due to lack of expertise of the teacher, or because it is difficult to fit these topics in the syllabus.

It has been conjectured that student-centred and inquiry-oriented teaching models make it easier to integrate generic skills. One reason is that these models put an emphasis on using diverse teaching and assessment methods. For example, projects, group work and self-assessment can be used to teach generic skills while the students are learning mathematics.

In this talk, I will discuss how student-centred and inquiry-oriented teaching models support integrating generic skills in mathematics teaching. In addition to theoretical literature, I will draw examples from a recent study we conducted on the historical development of the Extreme Apprenticeship method at the University of Helsinki.

On Solution of Elliptic Partial Differential Problems on Random Domains

Harri Hakula
Department of Mathematics and Systems Analysis, Aalto University

Modeling manufacturing imperfections is a classical engineering problem involving random domains. There are many computational approaches depending on the characteristics of the domain perturbations. For small perturbations, the *perturbation technique* of [1] is sufficient. For large deviations within simple geometric configurations, the *domain mapping* approach has been applied successfully, see [2]. It transfers the shape uncertainty onto a fixed reference domain and in the context of the standard Poisson problem, the randomness appears in the diffusion coefficient while the domain is fixed. The third approach, the focus in this talk, lets the discretisation of the domain account for the domain perturbations directly. If the sufficient software technology is readily available, this can be accomplished both in intrusive (Galerkin) and non-intrusive (collocation, QMC) settings.

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Surrogates in PDE-Constrained One-Shot Optimization under Uncertainty

Philipp A. Guth

Many problems in science and engineering can be formulated as optimization problems constrained by partial differential equations (PDEs) with random coefficients. Thereby the input random field is typically parametrized by a countably infinite number of parameters, and hence a very large number of PDE-solves is needed to obtain an accurate and robust solution of the underlying optimization problem.

Our approach replaces the computationally intensive solution of the parametric PDE by a surrogate, e.g., a neural network, which is learned simultaneously in a one-shot sense when solving the optimal control problem. We reformulate the problem as a penalized empirical risk minimization problem for which we provide a consistency analysis in terms of increasing sample size and increasing penalty parameter. To solve the resulting problem, we suggest a stochastic gradient method with adaptive control of the penalty parameter and prove convergence under suitable assumptions on the surrogate model. Numerical experiments illustrate the results for linear and nonlinear surrogate models.

High-dimensional kernel interpolation over lattice point sets with application to uncertainty quantification

Vesa Kaarnioja
LUT University

We describe a fast method for solving elliptic PDEs with uncertain coefficients using kernel-based interpolation over an integration lattice point set [1]. By representing the input random field of the system using the model proposed in [2], in which a countable number of independent random variables enter the random field as periodic functions, it is shown that the kernel interpolant can be constructed for the PDE solution (or some quantity of interest thereof) as a function of the stochastic variables in a highly efficient manner using fast Fourier transform. The method works well even when the stochastic dimension of the problem is large, and we prove rigorous error bounds which are independent of the stochastic dimension of the problem. The theoretical rates are assessed in a series of numerical experiments.

This talk is based on joint work with Yoshihito Kazashi, Frances Kuo, Fabio Nobile, and Ian Sloan.

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Dimensions of families of sets with applications to team semantics

Kerkko Luosto
Tampere University

Many modern fields of logic use different kinds of logical measures to determine complexity of various mathematical objects. As regards to logics with team semantics, there are several logical resources related to the syntactical complexity of formulas that can be used to that end. However, in our talk we consider semantic complexity: To each formula and structure, we relate the family of teams that satisfy the formula. We measure the complexity of such a family by three dimensions which we call upper dimension, dual upper dimension and cylindrical dimension of the family. Varying the finite structure, this gives rise to three dimension functions of the formula. In this setting, the complexity of a formula is measured by the growth class of the dimensional function under consideration. We study the properties of the three dimensional concepts, and show that they play well together with the logical constructs. As a result, we get a correspondence between semantic and syntactic concepts: if a formula is built using only certain k-ary atoms, this reflects to the growth class of the dimensional functions. Some hierarchy results are immediate.

This is joint work with Lauri Hella (Tampere University) and Jouko Väänänen (University of Helsinki).

Handling boolean events consistently

Daniele Mundici
Department of Mathematics and Computer Science, University of Florence, Italy

In his 1854 investigation on the laws of thought, Boole writes:

> ...the object of the theory of probabilities might be thus defined. Given the probabilities of any events, of whatever kind, to find the probability of some other event connected with them. [1, §XVI, 4, p.246]

This generalizes the familiar problem of logic consequence, independence, inconsistency: Given truth-values in $\{0, 1\}$ of any events, of whatever kind, to find the truth-values of some other event connected with them. Today Boole's problem is revived in the optimization version of the PSAT problem, [3], generalizing the satisfiability problem SAT. A contribution to Boole's problem was given by de Finetti in 1932 with his inconsistency (also known as "Dutchbookability", or "incoherence") criterion and theorem, [2]. When probability assessments take values in $\{0, 1\}$ we recover logical inconsistency. As noted by de Finetti himself, his theorem is powerful enough to imply the additivity *axiom* for disjunctions of incompatible events. Recent results [5] show that also the product law in the *definition* of independent events is a consequence of de Finetti's theorem. In 1905 Hilbert noted:

> Wir fassen das einfach als Definitionen auf, wiewohl im gegenwärtigen Zustande der Entwicklung besonders die Bezeichnungen "Axiom" und Definition noch etwas durcheinandergehen. (We simply take this as definitions, although in the present state of development, especially the terms axiom and definition are still a bit confused.) [4, p.168]

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Ordered fragments of first-order logic

Reijo Jaakkola
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The study of the computational properties of fragments of first-order logic is an active research area, which is motivated by the general observation that most of the logics used in computer science applications, such as the description logics, can be translated into first-order logic. The main goal of this area is to discover expressive fragments which have nice computational properties.

A fragment of first-order logic is called ordered if, roughly speaking, the syntax of the fragment restricts permutations of variables (with respect to some ordering of the set of all available variables) and the order in which the variables are to be quantified. Their study has been mainly motivated by the observation that they tend to have decidable satisfiability problems.

The two most well-studied ordered fragments are the ordered logic and the fluted logic. The purpose of this presentation is to present some recent results on the complexities of the satisfiability problems of these fragments and their syntactical variations.

This talk is based on the article [1].

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Demand and Supply Elasticities in the Finnish Roundwood Markets in 1995-2017

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Using theoretical framework of utility maximizing NIFP owner and profit maximizing forest enterprise, the supply and demand functions and corresponding elasticities of six main roundwood assortments traded in the Finnish roundwood markets were estimated. The empirical results based on the simultaneous equations method together with instrumental variables and annual data for 1995–2017 emphasise the importance of current price and price expectation mechanism on wood supply decisions. The stumpage price is important determinant for the demand for softwood sawlogs while the stumpage prices are not statistically relevant factors for the demand for pulpwood. The results also reveal that the transition mechanism of the change of final product's prices is transmitted more easily to demand for sawlogs and pine pulpwood than for the demands for spruce and birch pulpwood. Surprisingly, the harvesting potential measured as maximum economic removal potential as a proxy for growing wood reserve turned out to be not significant explanatory factor for wood supply decisions. Also, roundwood imports turned out to be statistically insignificant regressors for the demand for domestic roundwood with the exception of pine pulpwood, whereas the forest taxation reform had no statistically significant effect on the supply of roundwood compared to the period after the reform.

Keywords: roundwood; demand; supply; elasticities; forest industry; structural changes; Finland

Estimation of impulse-response functions with dynamic factor models: a new parameterization

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We propose a new parametrization for the estimation and identification of the impulse-response functions (IRFs) of dynamic factor models (DFMs). The theoretical contribution of this paper concerns the dynamic identification problem of the IRFs, which are not identified without further restrictions. We show how the minimal identifying conditions proposed by Bai and Wang (2015) are nested in the proposed framework and can be further augmented with overidentifying restrictions leading to efficiency gains. The current standard practice for the IRF estimation of DFMs is based on principal components, compared to which the new parametrization is less restrictive and allows for modelling richer dynamics. As the empirical contribution of the paper, we develop an estimation method based on the EM algorithm, which incorporates the proposed identification restrictions. In the empirical application, we use a standard high-dimensional macroeconomic dataset to estimate the effects of monetary policy shocks. We estimate a strong reaction of the macroeconomic variables, while the benchmark models appear to give qualitatively counterintuitive results. The estimation methods are implemented in the accompanying `R` package.

Similarity-augmented structural vector autoregression: The effects of forward guidance shocks in different monetary policy conditions

Visa Kuntze* Markku Lanne† Henri Nyberg‡

We develop a similarity-based structural vector autoregressive (SVAR) model using the similar clusters of data relevant for the prevailing initial macroeconomic conditions of interest. Our computationally attractive simple model enables us to uncover time-varying effects of structural economic shocks in a flexible manner in relevant local environments instead of relying on a model estimated from the entire sample period. Our empirical results show that the dynamic effects of forward guidance shocks are generally dependent on monetary policy stance and typically negligible for output and inflation.

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Parallel sessions C

15 Analytic Number Theory (chair: Joni Teräväinen)

Olli Järviniemi: *Composite values of shifted exponentials*

Jesse Jääsaari: *On fundamental Fourier coefficients of Siegel cusp forms of degree* 2

Yu-Chen Sun: *On the short interval version of the L_1 norm of the exponential sum defined with the Mobius function*

16 Symbolic Dynamics 1: Undecidability (chair: Jarkko Kari)

Markus Whiteland: *Some decidable properties of discrete time linear dynamical systems*

Ilkka Törmä: *Countable shift spaces*

Ville Salo: *Natural groups with undecidable conjugacy problem*

17 Geometric Analysis and PDEs 1 (chair: Ekaterina Mukoseeva)

Damian Dąbrowski: *Vitushkin's conjecture and sets with plenty of big projections*

Yawen Feng: *Second order regularity of solutions to parabolic p-Laplace equation*

Giacomo Del Nin: *Motion of surfaces driven by rough vector fields*

18 Mathematical physics 1 (chair: Alex Karrila)

Kalle Koskinen: *Metastates of the mean-field spherical model in a random external field*

Marina A. Ferreira: *Coagulation equations for open systems*

Jonas M. Tölle: *Stochastic pressure equation in enhanced geothermal heating*

19 Authentic assessment in mathematics: A panel discussion (chair: Juuso Nieminen)

The session is a panel discussion about assessment of mathematics. The participants in the panel discussion are Daniel Reinholz, Paola Iannone and Jussi Kangas, representing both teachers and researchers in university mathematics education.

20 Industrial Mathematics in Europe – case studies (chair: Esko Turunen)

Zoltan Horváth: *Model reduction for numerical simulation of compressible fluids and its application to urban air pollution simulation*

Kees Vuik: *From Theory to Practice: Scalable Iterative Solvers for the Helmholtz Equation*

Manuel B. Cruz: *Using Mathematical Technology to Improve Industry Competitiveness: An Example from the Automotive Sector*

21 Complex systems (chair: Ion Petre)

Andrzej Mizera: *Divide & Control: An Efficient Decomposition-based Approach towards the Control of Asynchronous Boolean Networks*

Sergiu Ivanov: *Sequential Reprogramming of Biological Network Fate*

Nicoleta Siminea: *A network-based approach to identify potentially active drugs in SARS-CoV-2 infection*

Composite values of shifted exponentials

Olli Järviniemi
University of Turku

Proving that there are infinitely many Mersenne primes and only finitely many Fermat primes is way beyond current methods. We consider the set of primes in exponentially sparse sequences, showing that the density of n for which $a^n - b$ is a prime is zero under GRH and a Brun–Titchmarsh inequality for the Chebotarev density theorem. The talk is based on a joint work with Joni Teräväinen.

On fundamental Fourier coefficients of Siegel cusp forms of degree 2

Jesse Jääsaari
Queen Mary University of London, United Kingdom

Let F be a Siegel cusp form of degree 2 and even weight. We consider the properties of the sequence of Fourier coefficients $a(F, S)$ at the matrices S such that $-4\det(S)$ is equal to a fundamental discriminant. We will describe results concerning sign changes, lower bounds, and upper bounds for these coefficients. This is joint work with Steve Lester and Abhishek Saha.

On the short interval version of the L_1 norm of the exponential sum defined with the Mobius function

Yu-Chen Sun
University of Turku

In 2001, Balog and Ruzsa proved the best possible bound for the L_1 norm of the exponential sum over r free numbers. As a corollary, they gave the the L_1 norm of the exponential sum defined with the Mobius function. In this talk, I will first introduce their work and then give the short interval version of their theorem, especially for the short interval version of the Mobius case. This is work in progress and our results might still change.

Some decidable properties of discrete time linear dynamical systems

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A linear dynamical system (LDS) (A, x) is the orbit of a point $x \in \mathbb{R}^d$ under a linear map $A\colon \mathbb{R}^d \to \mathbb{R}^d$. In this talk, I will discuss some decision problems related to LDS. In particular, we consider certain reachability problems of LDS. The setting is: given a LDS (A, x) and a semi-algebraic target set $T \in \mathbb{R}^d$, does the LDS intersect T? We discuss the borderline between hard instances and decidable instances (e.g., we give links to the Skolem problem, which asks: is it decidable whether a linear recurrence sequence contains a 0—an open problem for several decades now). We show that the problem is decidable when T has low dimension (as a semi-algebraic set). The methods used in the decidable cases quickly give rise to symbolic dynamical systems, whose properties we are able to analyse suitably.

Moving from reachability to more complicated properties, one might be interested in analysing the behaviour of LDS using logical formulas over a finite set of predicates on the variables. In this direction, I will briefly mention positive results on the Monadic Second Order model-checking problem for LDS with semialgebraic predicates.

Talk based on

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Countable shift spaces

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Let G be a finitely generated group, and consider colorings of G with a finite set A of colors. We denote by A^G the set of all such colorings and equip it with the product topology. The group G acts on A^G by translations $(g \cdot x)_h = x_{g^{-1}h}$, which are homeomorphisms. In this way the pair (A^G, G) forms a dynamical system. In the case $G = \mathbb{Z}$ we have two-way infinite sequences of colors which are shifted to the left or right, and in the case $G = \mathbb{Z}^2$ we have colorings of the infinite discrete plane. Topologically closed G-invariant subsets of A^G are called *G-shift spaces*.

As a concrete example, take $A = \{0, 1\}$, $G = \mathbb{Z}^2$, and define $X \subset A^{\mathbb{Z}^2}$ by requiring that for all $x \in X$ and $\vec{v} \in \mathbb{Z}^2$, if $x_{\vec{v}} = 1$, then $x_{\vec{v}+(0,1)} = x_{\vec{v}+(1,0)} = 0$. In other words, no two 1-symbols can lie in adjacent coordinates. Then X is a shift space, and it is a *shift of finite type (SFT)*, which means a shift space defined by bounded-range constraints. A *sofic shift* is obtained from an SFT by applying a symbol-to-symbol map. For example, the set $Y \subset \{0, 1\}^{\mathbb{Z}}$ of those sequences that contain at most one 1 is a one-dimensional sofic shift, since it can be obtained from the shift of finite type $Z \subset \{0, 1, 2\}^{\mathbb{Z}}$ with allowed length-2 patterns 00, 01, 12, 22 by the symbol-to-symbol map $0, 2 \mapsto 0$; $1 \mapsto 1$. We call Z an *SFT cover* of Y.

We are interested in shift spaces, in particular shifts of finite type and sofic shifts, that contain only countably many colorings. In the one-dimensional case $G = \mathbb{Z}$, they have a simple characterization: they are exactly those that have zero topological entropy. Their configurations consist of long periodic segments bordered by a bounded number of short period-breaking patterns. For $G = \mathbb{Z}^2$ this is no longer true, as countable SFTs form a proper subset of the zero-entropy ones, and their topological structure can be very intricate. However, all one-ended groups G enforce some structural constraints of their own: if $X \subset A^G$ is a countably infinite SFT, there exists $x \in X$ such that $\{1_G\} \neq \mathrm{Stab}_G(x) \neq G$. For $G = \mathbb{Z}^2$, this means that some $x \in X$ is periodic, but not in every direction. Some groups, such as free groups and Tarski monsters, admit no infinite countable SFTs, and in general the structure of countable G-SFTs depends heavily on the geometry of G.

In the more concrete context of multidimensional shift spaces, it is an important open question whether every sofic shift admits an SFT cover with the same topological entropy, as is the case for $G = \mathbb{Z}$. In particular, does every zero entropy sofic shift admit an SFT cover with zero entropy? It turns out that a countable analogue of this problem has a negative answer: there exist countable two-dimensional sofic shifts whose SFT covers are all uncountable. We present an example of such a sofic shift as well as partial results towards a characterization of those sofic shifts that do admit countable SFT covers.

The main tools in this research direction are point-set topology (in particular the concept of Cantor-Bendixson rank), geometric group theory (the geometric properties of G influence the existence and structure of countable G-SFTs) and techniques from discrete tiling theory (constructions of tilings with embedded computation, usually in the form of counter machines).

Natural groups with undecidable conjugacy problem

Ville Salo

In 1911, Dehn formulated three decidability questions for groups: the word problem, the conjugacy problem, and the isomorphism problem. All three were proven to be algorithmically undecidable in the 50's, even for finitely-presented groups. The undecidability proofs are computability-theoretic in nature: one builds groups whose presentation somehow encodes universal computation.

Here, we concentrate on the conjugacy problem. To our knowledge, in all groups that have been previously studied *for their own sake*, conjugacy has either been proven decidable, or the status of this problem is unknown. Thus, one may get the impression that the undecidability of conjugacy is just logicians' word play, and conjugacy is decidable for groups that "arise in nature".

In this talk, we show that this hypothetical impression is incorrect, and present two natural examples of groups with undecidable conjugacy problem. The groups are natural in the sense that they can be defined in a matter of minutes, without any discussion of Turing machines or computation. Both were also originally defined and studied out of general interest without the conjugacy problem in mind.

Specifically, we show that the (simple, finitely presented) Brin-Thompson 2V has undecidable conjugacy problem, and that the (residually finite, infinitely generated) group of reversible cellular automata has "eventually locally undecidable conjugacy problem", meaning that conjugacy is undecidable in all finitely-generated subgroups containing a certain (very simple) set of generators. The first example solves a question of Belk and Bleak, and the latter a question of Jalonen and Kari.

Vitushkin's conjecture and sets with plenty of big projections

Damian Dąbrowski
University of Jyväskylä

We say that a compact set $E \subset \mathbb{C}$ is *removable for bounded analytic functions* if for any open set $U \supset E$ all bounded analytic functions $f : U \setminus E \to \mathbb{C}$ admit an analytic extension to U. In the 60s Vitushkin conjectured that a set is removable if and only if it has negligible projections (i.e., almost every projection of E has zero length).

In the case of sets with σ-finite length Vitushkin's conjecture has been shown to be true by G. David in 1998. On the other hand, the conjecture fails for 1-dimensional sets with non-σ-finite length: in 1988 P. Jones and T. Murai constructed a set with negligible projections which is non-removable for bounded analytic functions.

In other words, one of the implications comprising Vitushkin's conjecture is false. It is still unknown whether the other implication is true. Suppose that E has non-negligible projections; does this imply that E is non-removable for bounded analytic functions? In this talk I am going to describe recent progress made on this question: if E has *plenty of big projections*, then E is non-removable for bounded analytic functions. Based on ongoing work with Michele Villa.

Second order regularity of solutions to parabolic p-Laplace equation

Yawen Feng
University of Jyväskylä

For the p-harmonic function u, one of the known estimate shows locally $W^{1,2}$-regularity for the nonlinear expression of the gradient $|Du|^\beta Du$ with some range of β. In this talk, we will extend this result to the parabolic p-Laplace equation

$$u_t = \Delta_p u = \operatorname{div}(|Du|^{p-2}Du),$$

with the sharp range $\beta > \frac{p-3}{2}$. This is joint work with M. Parviainen (University of Jyväskylä) and S. Sarsa (University of Helsinki).

Motion of surfaces driven by rough vector fields

Giacomo Del Nin
University of Warwick, United Kingdom

The theory of flows of vectorfields is a classical topic which is of key importance to understand the solutions of two PDE's of wide applicability:

- the *continuity equation*
$$\partial_t u_t + \operatorname{div}(\mathbf{b}\, u_t) = 0; \tag{CE}$$
- the *transport equation*
$$\partial_t u_t + \mathbf{b} \cdot \nabla u_t = 0. \tag{TE}$$

Given a vectorfield $\mathbf{b} : \mathbb{R}^d \to \mathbb{R}^d$, both equations model in different ways the notion of a family of time-dependent functions u_t which move according to the driving vectorfield $\mathbf{b}$.

While the theory for Lipschitz vectorfields $\mathbf{b}$ dates back to more than a century ago, more recent developments led by DiPerna-Lions and Ambrosio extended many results to the class of Sobolev and BV vectorfields. All these theories address the case where solutions u_t are sought in the class of functions, or at most in the class of measures.

In this seminar I will talk about recent efforts, motivated by the modeling of defects in plastic materials, towards extending the previous theory to the case when the unknown T_t are *k-currents* in $\mathbb{R}^d$, namely, generalised surfaces of dimension k. The equation involves the Lie derivative and takes the form
$$\partial_t T_t + \mathcal{L}_{\mathbf{b}} T_t = 0.$$
From this point of view, (CE) corrseponds to the case of 0-currents, while (TE) to the case of d-currents. I will explain the main challenges this problem presents and some results based on an ongoing research with Paolo Bonicatto and Filip Rindler (University of Warwick).

Metastates of the mean-field spherical model in a random external field

Kalle Koskinen
University of Helsinki

Infinite volume Gibbs states are probability measures obtained by taking limits of finite volume Gibbs states which are probability measures related to the canonical ensemble. For the mean-field spherical model, one can naturally introduce a random external field to the Hamiltonian which changes the original deterministic finite volume Gibbs states to random finite volume Gibbs states which are now random probability measures. In this talk, we will present a wide variety of issues and their resolutions associated with taking the infinite volume limits of these model specific random finite volume Gibbs states. In doing so, we will discuss the concept of chaotic size dependence and introduce the joint measure, the Aizenman-Wehr metastate, and the Newman-Stein metastate.

Coagulation equations for open systems

Marina A. Ferreira
University of Helsinki

Smoluchowski's coagulation equation is a classical mean-field model describing the evolution of the size distribution of particles which grow due to the collision and subsequent merging with each other. This model is widely used in the study of the growth of aerosols in the atmosphere. We focus on the analysis of coagulation equations in the presence of a source of particles. We study existence and non-existence of stationary (non-equilibrium) solutions for a large class of coagulation kernels, as well as the self-similar behaviour of time-dependent solutions. In the more general case where the composition of the particles is also taken into account, we will see that stationary solutions exhibit an interesting universal property: the mass localizes in a one-dimensional manifold as the size increases. Joint work with: Eugenia Franco, Jani Lukkarinen, Alessia Nota, Juan Velázquez.

Stochastic pressure equation in enhanced geothermal heating

Benny Avelin[*], Tuomo Kuusi[†], Patrik Nummi[†],
Eero Saksman[†], *Jonas M. Tölle*[‡], Lauri Viitasaari[*]

Motivated by the St1 Deep Heat project in Espoo, we are interested in modeling the high pressure creeping water-flow in crustal rock that occurs in enhanced geothermal heating (EGH). The EGH system consists of two 7-km-deep boreholes into the basement rock of the Nordic countries. Water is pushed into one hole and creeps through the basement rock into the second hole, heated by the geothermal heat in the process, such that the hot water can be extracted for energy. Understanding the fluid flow is essential for understanding the heat extraction.

From Darcy's law, describing creeping water-flow in porous media, we deduce the model for the pressure of the flow as an elliptic PDE depending on the local permeability κ. Geophysical modeling suggests that the porosity field can be expressed by a log-correlated Gaussian random field βX, where $\beta < \sqrt{d}$ and d is the spatial dimension. This results in the following stochastic PDE on a smooth domain

$$-\operatorname{div}[e^{\diamond \beta X} \diamond \nabla u] = 0,$$

where the Wick exponential term $\kappa = e^{\diamond \beta X}$ models the (random) permeability. The pressure gradient is renormalized by a Wick product. With the help of elliptic regularity theory and tools from the theory of Gaussian noise (that is, the S-transform), we shall define our notion of a solution to the above equation and establish its existence and uniqueness.

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Authentic assessment in mathematics: A panel discussion

The session is a panel discussion about assessment of mathematics. Recently, assessment research has been increasingly interested in authentic, disciplinary assessment practices. For example, 'authentic' practices would differ between medical education and teacher education, reflecting the practices that doctors and teachers use in their future professions. But what kinds of assessment practices are authentic to mathematics? What kind of assessment would reflect the authentic practices of mathematicians? The participants in the panel discussion are Dr Daniel Reinholz, Dr Paola Iannone and Dr Jussi Kangas, representing both teachers and researchers in university mathematics education. The panel will discuss student-centred assessment practices (e.g. self-assessment, peer-assessment, dialogic feedback practices) and their relevance from the viewpoint of authenticity.

Model reduction for numerical simulation of compressible fluids and its application to urban air pollution simulation

Zoltán Horváth, Mátyás Constans
Széchenyi István University, Győr, Hungary

Fast and accurate computational simulation of urban air pollution propagation for one day or a longer period like one full year is an important tool for environmental policy makers. However, application of a numerical software with the unsteady Navier–Stokes equations on fine 3D unstructured meshes of the large and complex city domain requests high performance computing infrastructure (e.g. supercomputers) whenever the user needs results within significantly shorter time than the simulated time (e.g. air flow simulation of a city for 1 day within 1 hour). To significantly reduce the computational time, some model order reduction methods, e.g. the projection-based proper orthogonal decomposition method (POD) could be used in principle, but several obstacles (e.g. small steps to avoid violation of inequality constraints at projections, costly projections) destroy performance.

In the talk, first we present the algorithmic details and numerical results with POD applied to the unsteady compressible Navier–Stokes equations with ideal gas EOS. The underlying full order solver is an inhouse upwind, 2nd order finite volume code operating on unstructured tetrahedral meshes with explicit time stepping. We obtained that our POD-implementation allows stable computations at huge CFL-numbers like 2,500 on meshes with 500,000 and 800,000 cells when the simplest explicit time-stepping scheme, the Explicit Euler method was used (c.f. the largest stable CFL number equals 0.85 for the original, full order method), and thus bring simulation time of a full day air flow computation to 1 minute only (on 1 GPU). Then, we pose a mathematical problem for the rigorous investigation of the stability of POD and we solve it for a special set of problems. For a tool, we shall formulate and prove a theorem on the cone-invariance property of the singular value decomposition of rectangular matrices.

From Theory to Practice: Scalable Iterative Solvers for the Helmholtz Equation

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At the heart of many applications ranging from medical and seismic imaging to telecommunications lies the Helmholtz equation. While the equation in itself, which is essentially the shifted Laplace equation, appears simple and elegant, retrieving accurate and scalable numerical solutions leads to a wide array of issues.

Due to the shift, the operator and consequently resulting discretized linear system matrix become indefinite. To ensure accurate solutions and the minimization of numerical dispersion, we are required to use relatively fine grids. Consequently, we end up with large linear system matrices, making the use of direct solution methods infeasible.

This shift represents the wave number and as the wave number increases, so do the number of iterations to reach convergence when using iterative solvers. In principle, this deteriorating convergence behavior can be attributed to near-zero eigenvalues of the preconditioned system.

For more than 15 years, the industry has relied on using the complex shifted Laplacian (CSL) as an effective preconditioner to accelerate the convergence. While this works efficiently for medium sized wave numbers, the number of iterations are still too high for practical applications and the problem sized become too large when we move to modern high-frequency problems.

So how can we design solvers which remain scalable both in terms of the computational complexity and the wave number? A natural choice would be to use multilevel methods. In this talk we will focus on both multilevel deflation and multigrid for these highly indefinite problems.

Getting multigrid solvers to work for the Helmholtz equation has been an open problem in applied mathematics for years. We present one of the first stand-alone multigrid solvers for the Helmholtz equation. We use standard smoothing techniques and do not require any restrictions on the number of grid points per wavelength on the coarse-grid. As a result we are able to obtain a full V- and W-cycle algorithm. The key features of the algorithm are the use of higher-order intergrid transfer operators, and a complex shift in the Jacobi smoothing operator.

Next we compare the performance and differences with a multilevel deflation approach. Here, we extend a two-level deflation method to a multilevel deflation method. By using similar higher-order deflation vectors, the near-zero eigenvalues of the the coarser grid operators remain aligned with the fine-grid operator keeping the spectrum of the preconditioned system fixed away from the origin.

Both proposed algorithms provide an important step towards the perpetuating branch of research in finding scalable solvers for wave propagation problems. We illustrate them in this talk by exploring both constant and variable heterogeneous model problems and addressing future challenges.

Using Mathematical Technology to Improve Industry Competitiveness: An Example from the Automotive Sector

Manuel B. Cruz
Polytechnic of Porto, Portugal

One of the main goals of Industrial Mathematics is to increase the impact of mathematics on innovations in key technologies and to foster the development of new modelling, simulation and optimization tools. Additionally, in some technology transfer activities, mathematicians are a great asset due to their capability to develop tailormade models, oriented to the industry specific needs.

However, the technology transfer between research groups and private companies in the context of Industrial Mathematics is usually unknow to the public. In this talk it will be presented a collaboration between a Mathematics research group, LEMA, and one of the biggest Portuguese companies in the automotive sector, Nors Group, addressing some of the most relevant factors that contributed to the success of this partnership initiated in 2014.

Divide & Control: An Efficient Decomposition-based Approach towards the Control of Asynchronous Boolean Networks

Andrzej Mizera
University of Luxembourg

We study the problem of computing a minimal subset of nodes of an asynchronous Boolean network that need to be simultaneously perturbed in a single-step to make the network dynamics eventually reach a target steady state (or attractor) from a given initial steady state. We refer to this problem as the minimal source-target control of Boolean networks. Due to the infamous phenomenon of state-space explosion, a direct global approach that performs computations on the entire network may not scale well. We hypothesise that efficient algorithms for the control of large Boolean networks need to exploit both the structures and the dynamics of the networks. Based on this, we devise a decomposition-based solution to the minimal source-target control which can be significantly faster than the existing approaches on large networks. By applying our algorithm to both real-life biological networks and randomly generated networks, we demonstrate the efficiency and effectiveness of our approach.

Sequential Reprogramming of Biological Network Fate

Sergiu Ivanov
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Network controllability is a major challenge in network medicine. The problem is to rewire the molecular networks to reprogram the cell fate. The reprogramming action is considered as a control usually performed once. However, in some cases, a therapy has to follow a time-scheduled drug administration protocol. Furthermore, some diseases are induced by a sequence of mutations leading to a sequence of actions on molecules. In this work, we extend the single control action method by investigating the sequential control of Boolean networks. We present a novel theoretical framework for formal study of control sequences, leading to algorithms resolving the PSPACE-hard problem of inferring minimal parsimonious control sequences under the synchronous dynamics.

A network-based approach to identify potentially active drugs in SARS-CoV-2 infection

Nicoleta Siminea
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National Institute of Research and Development for Biological Sciences, Romania

To find potentially active drugs in SARS-CoV-2 infection, we studied directed protein-protein interactions networks. In order to build such networks, we considered the 200 highest ranked genes obtained as required host factors in SARS-CoV-2 infection by Daniloski and his collaborators, but also the proteins on which approved or investigational drugs act. In addition to this data, we used information from other databases about proteins interactions to build our networks. Then we applied network controllability and found several proteins and drugs that could target them, as well as paths to control the restrictive factors. The drugs obtained are partially included in clinical trials, but also in other bioinformatics studies. Moreover, the network controllability allowed us to identify drug combinations that could be useful in the disease.

This presentation is based on joint work "Network analytics for drug repurposing in COVID-19" with Victor Popescu, Jose Angel Sanchez Martin, Daniela Florea, Georgiana Gavril, Ana-Maria Gheorghe, Corina Ițcuș, Krishna Kanhaiya, Octavian Pacioglu, Laura Ioana Popa, Romică Trandafir, Maria Iris Tușa, Manuela Sidoroff, Mihaela Păun, Eugen Czeizler, Andrei Păun, Ion Petre.

Parallel sessions D

22 Graph Theory (chair: Riikka Kangaslampi)

Henri Hansen: *Misuse of information in Insiders' social network*
Ragnar Freij-Hollanti: *Derived matroids of graphs*
Riikka Kangaslampi: *Ollivier–Ricci curvature on graphs*

23 Symbolic Dynamics 2: Tilings (chair: Jarkko Kari)

Svetlana Puzynina: *On small pattern abelian complexity of two-dimensional words*
Etienne Moutot: *Algebraic Tools for Nivat's Conjecture*
Rachel Greenfeld: *Decidability and periodicity of translational tilings*

24 Geometric Analysis and PDEs 2 (chair: Ville Tengvall)

Vesa Julin: *p-Harmonic coordinates for Hölder metrics and applications*
Saara Sarsa: *Second order Sobolev-regularity of p-harmonic functions*
Aleksis Koski: *Homeomorphic Extension Problems in Geometric Analysis*

25 Mathematical physics 2 (chair: Alex Karrila)

István Prause: *The five-vertex model*
Joona Oikarinen: *Stress-Energy Tensor in Liouville Conformal Field Theory*
Shinji Koshida: *The quantum group dual of the first-row modules for the generic Virasoro VOA*

26 Mathematical systems and control theory (chair: Mikael Kurula)

Thavamani Govindaraj: *Robust output tracking of a flexible satellite*
Daniele Cannarsa: *Approximately controllable finite-dimensional bilinear systems are controllable*
Lassi Paunonen: *Non-Uniform Stability of Damped Hyperbolic PDEs*

28 Mathematical logic 3 (chair: Kerkko Luosto)

Jouko Väänänen: *Towards internal categoricity*
Åsa Hirvonen: *Ehrenfeucht–Fraisse games on metric structures*
Erkko Lehtonen: *Associative spectra of groupoids*

29 The state of industrial mathematics in Finland. Panel discussion (chair: Esko Turunen)

Mikko Sauni: *Mathematics in railway asset management*
Panel discussion, chairs Zoltan Horváth (European Industrial Mathematics council) and Esko Turunen (Mathematical Modeling Network in Finland)

Misuse of information in insiders' social network

Henri Hansen
Tampere University

Juho Kanniainen
Tampere University

Insider trading is a real problem; for example, it has been documented that in the US, independent directors earn on average 4.87% more than the market, which is statistically highly significant and directors' dealings are found to be informative in European stock markets, too. While it has been extensively studied how insiders trade on their own company's securities, there is very limited knowledge about to which extent sensitive private information is used by external traders that have social connections to insiders.

Our research aims to identify such outsiders, i.e. persons who are not insiders of a company but have direct or indirect connections to the companies through insiders' social network. Our main objective is to find whether the investors' location in the the observable social network of insiders explains their returns around company announcements. In particular, the question is whether investors' trading performance before nonscheduled company announcements can be explained by their connections to the source of information (i.e. to the insiders of the company of which securities she trades).

Our data contains a certain (here undisclosed) stock market with companies and their board members at given dates, alongside trades on stocks of the said companies during a certain time interval. The data indicates that excess profits are indeed made by some – but not all – persons in the network. We consider two models that attempt to explain how the information has been passed to the persons that make such excess profits.

Firstly, the so-called *independent cascade* model, which models the flow of information from the insiders of a given company. This is a one parameter model, where the probability of information transmission is uniform. We try to find the parameter of this model via repeated Monte Carlo simulation, by minimizing a simple loss function.

Secondly, we consider a regression model in which we try to predict the probability of making excess profit around announcemnt dates, using measures such as the so-called betweenness centrality of a node, distance from the insiders in the network, and similar measures.

Our preliminary results indicate that a persons status in the graph does indeed predict their probability of making profitable trades.

Derived matroids of graphs

Ragnar Freij-Hollanti

It is well known that the edges of an arbitrary undirected graph form the ground set of a binary matroid, the circuits of which correspond to cycles in the graph. For a general matroid M, a problem that goes back to Crapo is to define a dependence structure – called a derived matroid – on the set of circuits of M. While this is straightforward for represented matroids, the derived matroid depends on the representation, and so is not intrinsically a matroid invariant. We propose a notion of a generic derived matroid, which can be defined for arbitrary matroids and maps weakly to any other derived matroid in the representable case. In the case of graphical matroids, we interpret our new definition graph-theoretically, and compare it to the binary derived matroid which is also naturally defined for graphical matroids.

This is joint work with Reline Jurrius and Olga Kuznetsova.

Ollivier–Ricci curvature on graphs

Riikka Kangaslampi, Tampere University

Curvature is a fundamental notion in the study of smooth Riemannian manifolds. This notion has been generalized in various ways from the smooth setting of manifolds to more general metric spaces. This talk considers the Ollivier-Ricci curvature in the discrete setting of combinatorial graphs. I will introduce the Ollivier–Ricci curvature and present a few examples, calculating the curvature numerically using the interactive web-application at https://www.mas.ncl.ac.uk/graph-curvature/. I will also present a classification of cubic graphs with girth five that have zero curvature, and briefly discuss results on the behaviour of the Ollivier–Ricci curvature under graph products.

On small pattern abelian complexity of two-dimensional words

Svetlana Puzynina (joint work with Nikolai Geravker)
Saint Petersburg State University, Russia

In this talk, we will discuss the relations between periodicity of two-dimensional words and their abelian (pattern) complexity. A pattern $\mathcal{P}$ in $\mathbb{Z}^n$ is the set of all translations of some finite subset F of $\mathbb{Z}^n$. An F-factor of an infinite word is a finite word restricted to F. Then the *pattern complexity* over a pattern $\mathcal{P}$ counts the number of distinct F-factors of an infinite word, for $P \in \mathcal{P}$. The notion of a classical *complexity* of a two-dimensional word w is a particular case of the pattern complexity for rectangular blocks, i.e. it is defined as a function $p_w(m,n)$ counting the number of distinct rectangular $m \times n$ blocks of a two-dimensional word. The famous Nivat's conjecture, 1997, states that Let w be a two-dimensional word. If there exists m, n such that $p_w(m,n) \leq mn$, then w has a periodicity vector. The notion of a complexity can be generalized in the abelian setting as follows. Two finite words are called abelian equivalent if for each letter of the alphabet, they contain the same numbers of occurrences of this letter. The *abelian pattern complexity* counts the number of F-factors up to abelian equivalence. For the abelian complexity for rectangular shapes, contrary to the one-dimensional case, there exist aperiodic words with abelian complexity 1 for some block sizes [1]. Further, for recurrent words abelian complexity cannot be bounded by 2 and moreover the abelian complexity at least 3 must be achieved for infinitely many block sizes [1]. For non-rectangular patterns, we characterize two-dimensional convex patterns with the following property: if abelian pattern complexity over a pattern $\mathcal{P}$ is equal to 1, then the word is fully periodic [2]. Similar result holds for a function on $\mathbb{Z}^2$ instead of a word and for constant sums instead of abelian complexity equal to 1. In dimensional 1, we characterize patterns for which there exist non-constant functions with constant sums.

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Algebraic Tools for Nivat's Conjecture

Etienne Moutot[1]

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Nivat's conjecture states that any coloring of $\mathbb{Z}^2$ with "not too many different patterns" must be periodic. The number of different colorings of rectangles of size $m \times n$ is called the *complexity* of the coloring (or configuration) and is denoted by $P(m, n)$. The precise conjecture is that if there exists $m, n \in \mathbb{N}$ such that $P(m, n) \leq mn$, then the configuration must be periodic. A configuration for which $P(m, n) \leq mn$ is called a *low complexity configuration*. Although simple to understand, this conjecture is still open since stated by Maurice Nivat in 1997.

Over the years, there have been numerous attempts to approach the conjecture. For example, it has been proved that if m is fixed equal to two or three, the conjecture holds (respectively by Sanders and Tijdeman [ST02], and Cyr and Kra [CK16]). It has also been proved that if the configuration has less than a fraction of mn instead of mn, the conjecture holds. The best result in this direction is due to Cyr and Kra who proved in 2015 that if $P(m, n) \leq \frac{mn}{2}$, then the configuration is periodic [CK15].

In 2015, Jarkko Kari and Michal Szabados developed powerful algebraic tools to work with low complexity configurations [KS15]. Using these, they were able to prove several results close to the conjecture. They proved for example that the conjecture holds if there exists infinitely many m, n such that $P(m, n) \leq mn$, or that any low complexity configuration can be decomposed into a sum of periodic ones.

With Jarkko Kari, we continued to work one these tools, and incorporated more dynamical ideas from Cyr and Kra. In particular we studied directions of determinism, which allowed us to prove that Nivat's conjecture holds for uniformly recurrent configurations, i.e. configurations with all of its patterns appearing uniformly in the configurations [KM20, KM21]. We also developed purely algebraic tools showing that a generalized version of Nivat's conjecture holds for configurations meeting a particular algebraic condition [KM19].

The first goal of this talk is to provide an overview of some of algebraic tools around Nivat's conjecture. Then, I will sketch the proof that the conjecture holds in the uniformly recurrent case.

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Decidability and periodicity of translational tilings

Rachel Greenfeld, UCLA

Let G be a finitely generated abelian group, and $F_1, ..., F_J$ be finite subsets of G. We say that $F_1, ..., F_J$ tile G by translations, if G can be covered by translated copies of $F_1, ..., F_J$, without any overlaps.

Given some finite sets $F_1, ..., F_J$ in G, can we decide whether they admit a tiling of G? Suppose that they do tile G, do they admit any periodic tiling? A well known argument of Hao Wang ('61) shows that these two questions are closely related. In the talk, we will discuss this relation, and present some results, old and new, about the decidability and periodicity of translational tilings. The talk is based on an ongoing project, joint with Terence Tao.

p-Harmonic coordinates for Hölder metrics and applications

Vesa Julin

In this talk I will discuss how one finds a local p-harmonic coordinate system in Riemannian manifold with Hölder continuous metric tensor. When $p = n$ this leads to a useful gauge condition for regularity results in conformal geometry. As an application, we obtain that any conformal mapping between manifolds with C^α metric tensors is $C^{1+\alpha}$ regular, and that a manifold with $W^{1,n} \cap C^\alpha$ metric tensor and with vanishing Weyl tensor is locally conformally flat if $n \geq 4$. This is a joint work with Tony Liimatainen and Mikko Salo.

Second order Sobolev-regularity of p-harmonic functions

Saara Sarsa, University of Helsinki

Let u denote a p-harmonic function and Du denote its gradient ($1 < p < \infty$). Being p-harmonic means that u solves the equation

$$\operatorname{div}(|Du|^{p-2}Du) = 0$$

in the weak sense. It is well known that the nonlinear transformation $|Du|^{\frac{p-2}{2}}Du$ of the gradient Du belongs to the Sobolev space $W^{1,2}_{\mathrm{loc}}$. In this talk I will discuss on some recent extensions of this result.

Homeomorphic Extension Problems in Geometric Analysis

Aleksis Koski, *Universidad Autónoma de Madrid*

One of the most fundamental problems in Geometric Analysis is to understand which properties of a boundary map allow for a homeomorphic extension with specific geometric and analytic properties. Classical results such as the Beurling-Ahlfors extension result or the Radó-Kneser-Choquet theorem constitute some of the basic building blocks needed to solve these problems in 2D space. In this talk, we will review the known planar theory of questions such as the *Sobolev Jordan-Schönflies problem* of extending a boundary map between Jordan domains as a Sobolev homeomorphism. Moreover, we discuss some first approaches in higher dimensions where many of the techniques crucial to the planar theory simply fail. This talk is based on joint work with Stanislav Hencl and Jani Onninen.

The five-vertex model

István Prause, University of Eastern Finland

The five-vertex model is a special case of the six-vertex model. It is a probability measure on monotone nonintersecting lattice path configurations on the square lattice where each corner-turn is penalised by a weight. It can also be viewed as an interacting (non-free fermionic) version of the well-known lozenge tiling model. I'll study the limit shape phenomenon for this model. That is, we are interested in the typical shape of configurations for large system size and fixed boundary conditions, such as the "boxed plane partition". I'll highlight the similarities and new features compared to the free fermion case. The talk is based on joint work with Rick Kenyon (Yale University).

Stress-Energy Tensor in Liouville Conformal Field Theory

Joona Oikarinen*

We describe the probabilistic construction of the path integral of Liouville theory, which is based on the Gaussian Free Field and Gaussian Multiplicative Chaos. The theory exhibits conformal symmetry, which we formulate in terms of the background metric of the theory, showing that the theory is a Conformal Field Theory. Finally, we explain how to define the Stress-Energy tensor in terms of the background metric, and how to derive the Conformal Ward identities for the correlation functions of the Stress-Energy tensor. These identities can be seen as a local formulation of conformal symmetry. Partly based on joint work with Antti Kupiainen.

*University of Helsinki

The quantum group dual of the first-row modules for the generic Virasoro VOA

Shinji Koshida, Department of Mathematics and Systems Analysis, Aalto University

In several examples it has been observed that a module category of a vertex operator algebra (VOA) is equivalent to a category of representations of some quantum group. In this talk, we discuss such a duality in the case of the Virasoro VOA at generic central charge. We do not address the category of all modules of the generic Virasoro VOA, but we consider the infinitely many modules from the first row of the Kac table. Building on an explicit quantum group method of Coulomb gas integrals, we give a new proof of the fusion rules and the analyticity of compositions of intertwining operators. Crucially, we prove the associativity of the intertwining operators among the first-row modules, and find that the associativity is governed by the $6j$-symbols of the quantum group $\mathcal{U}_q(sl_2)$. This talk is based on a joint work with Kalle Kytölä.

Robust output tracking of a flexible satellite

Thavamani Govindaraj*

In this talk, we will focus on a recent result on robust output tracking of a flexible satellite. We consider a satellite model that is composed of two flexible solar panels and a rigid center body. We will discuss exponential stability of the model using linear semigroup theory. In addition, we will consider output tracking problem for the satellite model. In this part, we will present a controller that achieves robust output tracking of the satellite model.

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Approximately controllable finite-dimensional bilinear systems are controllable

Daniele Cannarsa*

In this talk we show that a bilinear control system is approximately controllable if and only if it is controllable in $\mathbb{R}^n \setminus \{0\}$. We approach this property by looking at the foliation made by the orbits of the system, and by showing that there does not exist a codimension-one foliation in $\mathbb{R}^n \setminus \{0\}$ with dense leaves that are everywhere transversal to the radial direction. The proposed geometric approach allows to extend the result to homogeneous systems that are angularly controllable. [Joint work with Mario Sigalotti.]

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Non-Uniform Stability of Damped Hyperbolic PDEs

Lassi Paunonen*

In this presentation we study the stability properties of hyperbolic linear partial differential equations, especially damped wave equations. A large class of such equations can be represented as abstract differential equations of the form

$$\ddot{w}(t) + Lw(t) + DD^*\dot{w}(t) = 0, \qquad w(0) = w_0, \quad \dot{w}(0) = w_1 \tag{1}$$

on a Hilbert space H. Here L is a positive and boundedly invertible operator on H and D is a bounded operator. In this presentation we study so-called *non-uniform stability* and *polynomial stability*. The characteristic feature of these stability types is that the solutions of (1) may decay at sub-exponential rates as $t \to \infty$, and these rates depend on the smoothness of the initial data. This kind of stability is especially encountered in multidimensional wave equations with partial or weak dampings. As our main results we introduce new sufficient conditions for the non-uniform and polynomial stability of (1) based on *observability properties* of the pair (D^*, L). We also apply our results for particular partial differential equations and discuss the optimality of the obtained degrees of stability. The presentation is based on the article:

R. Chill, L. Paunonen, D. Seifert, R. Stahn and Y. Tomilov. Non-uniform stability of damped contraction semigroups, *Analysis & PDE*, accepted for publication. arxiv.org/abs/1911.04804

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Towards internal categoricity

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I will discuss the role of second order logic and set theory in the foundations of mathematics. In particular I will focus on the categoricity of some important foundational second order theories. Finally, I will present some results about so-called internal categoricity both in the second order and in the first order case.

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Ehrenfeucht-Fraïssé games on metric structures

Åsa Hirvonen, University of Helsinki

In model theory Ehrenfeucht-Fraïssé games are a way of grading the similarities of structures of a given vocabulary. One player tries to prove the structures different while the other tries to show they are equivalent. The longer the second player can survive the game, the more similar the structures are. If the game is played on countable structures and the second player can survive ω moves, then the structures are isomorphic. The game also gives rise to so called Scott sentences – sentences of infinitary first order logic that determine the isomorphism class of a given countable structure.

When looking at metric structures, i.e., structures consisting of a metric space with continuous functions on them, one is usually not interested in countable structures but separable ones. The game then necessarily changes in character, as one cannot play through all elements with countably many moves, but only a dense set. This requires a change to a logic allowing for approximations, as small errors must be allowed for in the moves. In this talk I will present a countable game that captures isomorphism between separable metric structures. In the game no single move corresponds to a final mapping, but the moves give better and better approximations of an isomorphism. The approximate nature of the game also allows for a wider range of isomorphisms to be considered, such as linear isomorphisms of Banach spaces. The infinite game also comes hand in hand with a corresponding dynamic Ehrenfeucht-Fraïssé game, giving rise to Scott sentences.

This is joint work with Joni Puljujärvi.

Associative spectra of groupoids

Erkko Lehtonen (Universidade Nova de Lisboa)

Various notions for quantifying the degree of (non)-associativity of binary operations have been proposed in the literature. One such notion is the associative spectrum, which is based on how many identities that are consequences of the associative law are satisfied by the operation or the corresponding groupoid. This concept was introduced by Csákány and Waldhauser [2], and it has appeared in the literature under a number of different names, such as "subassociativity type" [1] and "the number of $*$-equivalence classes of parenthesizations" [3, 4].

A *bracketing* of n variables is a groupoid term obtained by inserting pairs of parentheses in a valid way into the string $x_1x_2\dots x_n$. The *associative spectrum* of a groupoid $\mathbf{A} = (A, \circ)$ is the sequence $(s_n(\mathbf{A}))_{n\in\mathbb{N}}$, where $s_n(\mathbf{A})$ is the number of distinct term operations induced on $\mathbf{A}$ by the bracketings of n variables. Intuitively, the faster the associative spectrum grows, the less associative the operation is considered. We clearly have $1 \le s_n(\mathbf{A}) \le C_{n-1}$, where C_{n-1} is the $(n-1)$-th Catalan number. If $\mathbf{A}$ is associative, then $s_n(\mathbf{A}) = 1$ for all n. A groupoid $\mathbf{A}$ is *antiassociative,* if $s_n(\mathbf{A}) = C_{n-1}$ for all $n \ge 2$.

In this talk, we present some recent results on the associative spectra of quasigroups and graph algebras [5, 6]. A *quasigroup* is a groupoid $(A, \circ)$ such that for all $a, b \in A$, there exist unique elements $x, y \in A$ such that $a \circ x = b$ and $y \circ a = b$. The *graph algebra* of a digraph $G = (V, E)$ is the algebra $\mathbb{A}(G) = (V \cup \{\infty\}; \cdot, \infty)$ of type $(2, 0)$, where ∞ is a new element distinct from the vertices, and $x \cdot y = x$ if $(x, y) \in E$, and $x \cdot y = \infty$ otherwise.

This is joint work with Tamás Waldhauser (University of Szeged).

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Mathematics in railway asset management

Mikko Sauni

Railways are subjected to recurring train loads and harsh environmental conditions. These cause incremental settlements in the railway track structure, which move the position of the rails. These settlements and consequential movements are not uniform, because the track structure varies along a track section, for example, due to elevation changes, bridges, and tunnels. As a result, the railway tracks become uneven. To ensure that the tracks are safe to operate on, track geometry measurements are conducted regularly using a specific track geometry measurement car. This measurement car provides data concerning irregularities that can be observed from the rail surface. If the track become too uneven, a tamping machine is used to lift the rails upwards while packing ballast bellow the tracks to create a smooth surface for the track.

The regularly conducted track geometry measurements can be combined to form time series data depicting the development in the unevenness of railway tracks. From this data, it is possible to detect problematic areas with high deterioration rates, past maintenance actions, and recurring defects. All this is vital information in explaining how the track structure has endured so far and how well the tracks can be maintained. In the past, these types of analyses have generally been made subjectively. In practice, this means that a person looks at graphs of different measurement runs and decides for him or herself if the areas are problematic and how maintenance has affected the area. Now though, data analytics have become more and more connected in track maintenance and a shift towards modelling track geometry deterioration based on measurement results is being taken.

Track geometry deterioration modelling begins with identifying maintenance actions from the measurement history. These are shown as improvements in the condition of the track geometry. The maintenance actions must be back calculated from the measurement history because no reliable repositories for past maintenance work are available. The simplest way to identify past maintenance is to set a threshold for improvements in the track geometry. However, as there is slight inaccuracy in the measurements, it can be sometimes difficult to tell what is caused by maintenance and by measurement inaccuracy. Therefore, past maintenance identification from track geometry history has been studied using different mathematical approaches. For example, a bachelor's thesis done at the Tampere University studied the use of mixture models in identifying tamping cycles.

After identifying the maintenance actions from the track geometry history, the track geometry deterioration between two maintenance actions is modelled. The behavior is generally linear, however, there is an initial settlement right after maintenance, which must be accounted for. Therefore, robust linear optimization was used for the linear models rather than simple linear regression, such as least squares. The robust linear optimization ignores the outliers caused by the initial regression to some degree without deleting the outlier data. This is especially important, as these outliers are real observations that are useful in assessing the track geometry history, but it is not desirable to have them influence the modelling results. The models are then used to describe the past behavior and produce projections for future development with measures for prediction uncertainty.

The next step and future direction in the research is to optimize maintenance actions. A practical example of this would be that the track geometry deterioration modelling has shown that there is five kilometers of track that should be renewed, but there are resources to renew only three kilometers of track. The optimization task is to select which three kilometers of the five should be renewed to achieve the highest overall condition with long-term effects. This is a complex task that deals with predictions and optimization simultaneously. However, it is also a concrete example of how mathematics can improve the way we maintain physical assets, such as railway structures.

Panel discussion on Industrial Mathematics in Finland

How can international co-operation accelerate it? With Zoltan Horvath (European Industrial Mathematics council), Esko Turunen (Mathematical Modeling Network in Finland) and Harry Länsipuro (TAU Innovation Services and Partnerships).