

# Inverse Days 2021

14.-16.12.2021, Tampere University

Book of Abstracts





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To our dearest conference participants and guests,

Inverse Days is the annual scientific workshop of the Finnish Inverse Problems Society (FIPS). This year's event is already 27th Inverse Days and the third one organized in Tampere. This year marks a return to a semi-normal workshop organized on site after last year's virtual conference, although the workshop has also many virtual participants and presentations and some pandemic regulations such as a recommendation to wear a face mask inside the venue.

This year's workshop has many invited speakers and wide international participation. The presentations cover a wide variety of topics from theoretical to applied inverse problems. There are Keynote sessions, up to four parallel sessions, a poster session, a Women in Inverse Problem meeting, and a Helsinki Deblur Challenge session.

We are grateful for the support provided by Tampere University in organizing this event and look forward to meeting scientists and students from all over the world in Tampere.

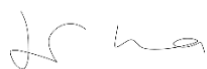
On behalf of the organizers,  
Sincerely,



**Sampsa Pursiainen**  
Associate Professor



**Pasi Raumon**  
Associate Professor



**Joonas Ilmavirta**  
Assistant Professor

## List of Participants

First name	Last name	Organisation
Taiwo	Adedipe	LUT University
Laia	Amoros	Finnish Meteorological Institute
Arttu	Arjas	University of Oulu
Antti	Autio	Aalto University
Katrine	Bangsgaard	Technical University of Denmark
Emilia	Blåsten	Aalto University
Valentina	Candiani	University of Genoa
Giovanni	Covi	University of Heidelberg
Jacques	Cuenca	Siemens Industry Software
Astrid	Dufaure	Institut Fresnel - Aix Marseille Université
Reda	Elkhadrawy	Tampere University Institut für Biomagnetismus und Biosignalanalyse, Münster
Tim	Erdbrügger	Institut Fresnel
Christelle	Eyraud	Tampere University
Fernando	Galaz Prieto	University of Sheffield
Adrien	Gallet	Aarhus University
Henrik	Garde	Utrecht University
Martin	Genzel	LUT University
Heikki	Haario	Finnish Meteorological Institute
Janne	Hakkarainen	Marquette University
Sarah	Hamilton	Aalto University
Antti	Hannukainen	University of Oulu
Andreas	Hauptmann	University of Helsinki
Tommi	Heikkilä	LUT University
Tapio	Helin	Finnish Meteorological Institute
Manu	Holmberg	University of Eastern Finland
Marzieh	Hosseini	Aalto University
Nuutti	Hyvönen	University of Eastern Finland
Niko	Hänninen	LUT University
Teemu	Härkönen	Institute for Biomagnetism and Biosignalanalysis, University of Münster
Malte	Höltershinken	University of Jyväskylä
Joonas	Ilmavirta	University of Helsinki
Bjorn	Jensen	Astroock Oy
Arto	Julkunen	University of Helsinki
Markus	Juvonen	LUT University
Vesa	Kaarnioja	University of Eastern Finland
Anna	Kaasinen	University of Eastern Finland
Jonna	Kangasniemi	University of Helsinki
Elli	Karvonen	Finnish Meteorological Institute
Anu	Kauppi	
Alexey	Kazarnikov	University of Heidelberg
Hanne	Kekkonen	Delft University of Technology
Mahnaz	Khalili	University of Eastern Finland
Eero	Koponen	University of Eastern Finland
Alexandra	Koulouri	Tampere University
Pu-Zhao	Kow	University of Jyväskylä
Antti	Kujanpää	University of Helsinki

## List of Participants

Antti	Kukkurainen	Finnish Meteorological Institute
Jaakko	Kultima	University of Oulu
Topi	Kuutela	Aalto University
Antti	Kykkänen	University of Helsinki
Joonas	Lahtinen	Tampere University
Matti	Lassas	University of Helsinki
Salla	Latva-Äijö	University of Helsinki
Piia	Lesonen	University of Eastern Finland
Ji	Li	National University of Singapore
Kristoffer	Linder-Steinlein	Technical University of Denmark
Boya	Liu	North Carolina State University
Stephanie	Lohrengel	University of Reims Champagne-Ardenne
Jinpeng	Lu	University of Helsinki
Timo	Lähivaara	University of Eastern Finland
Shiqi	Ma	University of Jyväskylä
Jan	Macdonald	Technische Universität Berlin
Anssi	Manninen	University of Oulu
Alexander	Meaney	University of Helsinki
Lauri	Mehtätalo	Natural Resources Institute Finland (Luke)
Reed	Meyerson	University of Helsinki
Antti	Mikkonen	Finnish Meteorological Institute
Rashmi	Murthy	University of Helsinki
Maximilian	März	Technische Universität Berlin
Keijo	Mönkkönen	University of Jyväskylä
Frank	Neugebauer	Tampere University
Angèle	Niclas	Ecole Centrale de Lyon
Matti	Niskanen	UEF
Jalo	Nousiainen	LUT University
Janne	Nurmela	Finnish Meteorological Institute
Janne	Nurminen	University of Jyväskylä
Medet	Nursultanov	University of Helsinki
Jonne	Nyberg	University of Jyväskylä
Lauri	Oksanen	University of Helsinki
Petri	Ola	University of Helsinki
Hanna	Partio	University of Helsinki
Annalisa	Pascarella	Institute of Applied Mathematics M. Picone, National Council of Research
Daniel M.	Pelt	Leiden University
Manuel	Pena	Universidad Politécnica de Madrid
Petteri	Piironen	University of Helsinki
Francesca	Pitolli	University of Roma 'La Sapienza'
Jenni	Poimala	University of Oulu
Leyter	Potenciano Machado	University of Jyväskylä
Sampsa	Pursiainen	Tampere University
Juha-Pekka	Puska	Aalto University
Narayan	Puthanmadam	Tampere University
Jesse	Subramaniyam	University of Cambridge
Aksel	Railo	University of Cambridge
Pasi	Rasmussen	Technical University of Denmark
Pasi	Raunonen	Tampere University
Siiri	Rautio	University of Helsinki
Atena	Rezaei	Tampere University
Lassi	Roininen	LUT University

## List of Participants

Rodrigo	Rojo Garcia	LUT University
Snizhana	Ross	University of Oulu
Andreas	Rupp	LUT University
Mikko	Räsänen	University of Eastern Finland
Teemu	Sahlström	University of Eastern Finland
Suman Kumar	Sahoo	University of Jyväskylä
Teemu	Salminen	University of Eastern Finland
Aleksi	Salo	LUT University
Mikko	Salo	University of Jyväskylä
Hjørdis Amanda	Schlüter	Technical University of Denmark
Abdu Mohammed	Seid	Bahir Dar University
Soumen	Senapati	TIFR-CAM, Bangalore
Angelina	Senchukova	LUT University
Samuli	Siltanen	University of Helsinki
Fernando	Silva de Moura	University of Helsinki
Sara	Sommariva	CNR-SPIN
Alberto	Sorrentino	Universita degli Studi di Genova
Liisa-Ida	Sorsa	Tampere University
Tomás	Soto	LUT University
Sebastian	Springer	University of Oulu
Miika	Suhonen	University of Eastern Finland
Anna	Suomenrinne-Nordvik	University of Helsinki
Ensio	Suonperä	University of Helsinki
Jarkko	Suuronen	LUT University
Tanja	Tarvainen	University of Eastern Finland
Theophil	Trippe	TU Berlin
Leo	Tzou	University of Sydney
Tuomo	Valkonen	University of Helsinki
Marko	Vauhkonen	University of Eastern Finland
Anton	Vavilov	Aalto University
Vincent	Verhoeven	Tampere University
Alessandro	Viani	University of Genova
Anne	Virkki	University of Helsinki
Antti	Voss	Kuopio University Hospital
Silja	Westphal Christensen	Technical University of Denmark
Ville-Veikko	Wettenhovi	University of Eastern Finland
		University of Münster, Institute for Biomagnetism and
Carsten	Wolters	Biosignalanalysis
Lauri	Ylinen	University of Helsinki
Yusuf Oluwatoki	Yusuf	Tampere University
Sen	Zou	Fudan University/University of Jyväskylä

**137 participants in total**

10:00–10:45 *Registration (1<sup>st</sup> floor lobby)*

10:45–11:00 *Conference Opening*

Morning session 1, Lecture hall A1	
11:00–11:45	Stephanie Lohrengel: Diffusive optical tomography in the neonatal head: a homogenized model for the cerebrospinal fluid layer
11:45–12:30	Christelle Eyraud: Microwave imaging from manufactured targets to small solar bodies

12:30–13:30 *Lunch (campus)*

12:30–14:30 *Board meeting (restaurant cabinet)*

	Parallel sessions, Lecture hall A1, Asteroid numerical modelling	Parallel sessions, Lecture hall A2a, Gaussian priors and Correction models	Parallel sessions, Lecture hall A3, Stroke classification	Parallel sessions, Lecture hall A4, Acoustic tomography
13:30–13:55	Liisa-Ida Sorsa	Hanne Kekkonen	Samuli Siltanen	Tanja Tarvainen
13:55–14:20	Astrid Dufaure	Andreas Hauptmann	Fernando Moura	Shiqi Ma
14:20–14:45	Yusuf Oluwatoki Yusuf	Jenni Poimala	Rashmi Murthy	Niko Hänninen

14:45–15:15 *Coffee break (portrait foyer, 2<sup>nd</sup> floor), Posters (on-site)*

Afternoon session 1, Lecture hall A1	
15:15–16:00	Matti Lassas: Inverse problems for a random walk on a finite graph
16:00–16:30	Anne Virkki: Inverse modeling of near-earth asteroids using ground-based planetary radar observations
16:30–17:00	Abdu Mohammed Seid: The spatiotemporal dynamics of water hyacinth over lake Tana of Ethiopia using hidden Potts prior and dynamic linear models

17:00–17:30 *Snacks and drinks*

17:30–19:00 *e-Posters*

19:00–20:30 *Reception at Café & Aula Toivo (campus)*

20:30– *Unofficial after-party at Restaurant Plevna*



8:45–9:00 *Registration (1<sup>st</sup> floor lobby)*

	Parallel sessions, Lecture hall A1, Bayesian EEG/MEG imaging and functional brain connectivity	Parallel sessions, Lecture hall A2a, Helsinki Deblur Challenge 2021	Parallel sessions, Lecture hall A3, Harmonic operators	Parallel sessions, Lecture hall A4, Stability and perturbations
9:00–9:25	Alberto Sorrentino	Ji Li	Boya Liu	Giovanni Covi
9:25–9:50	Alessandro Viani	Daniel Pelt	Jaakko Kultima	Pu-Zhao Kow
9:50–10:15	Sara Sommariva	Theophil Trippe	Suman Sahoo	Leyter Potenciano- Machado

10:15–10:45 *Coffee break (campus), Posters (on-site)*

Morning session 2, Lecture hall A1	
10:45–11:15	Atena Rezaei: Reconstructing subcortical and cortical activity of somatosensory via inverse modelling techniques utilizing SEP datasets
11:15–12:00	Carsten H. Wolters: New non-invasive multimodal neuroimaging and neurostimulation methods for improved diagnosis and therapy in refractory focal epilepsy

12:00–13:00 *Lunch (campus)*

12:30–13:30 *Women in Inverse Problems (restaurant cabinet)*

	Parallel sessions, Lecture hall A1, FEM in EEG/MEG	Parallel sessions, Lecture hall A2a, Electromagnetic wave data	Parallel sessions, Lecture hall A3, Manifolds I	Parallel sessions, Lecture hall A4, Tomography and resources
13:30–13:55	Tim Erdbrügger	Marzieh Hosseini	Joonas Ilmavirta	Tommi Heikkilä
13:55–14:20	Frank Neugebauer	Snizhana Ross	Janne Nurminen	Angele Niclas
14:20–14:45	Malte Höltershinken	Manuel Pena	Antti Kykkänen	Alexander Meaney

14:45–15:15 *Coffee break (portrait foyer, 2<sup>nd</sup> floor), Posters (on-site)*

Afternoon session 2, Lecture hall A3	
15:15–16:00	Francesca Pitolli: Randomized sampling methods for linear inverse problems
16:00–16:30	Sarah Hamilton: Fast 3D CGO-based reconstruction for absolute electrical impedance tomography on experimental data from ACT5
16:30–17:00	Annalisa Pascarella: An in-vivo comparison of source localization methods

17:00–18:00 *FIPS meeting (Lecture hall A3)*

19:00–21:00 *Dinner at Restaurant Tampella*

8:45–9:00 *Registration (1<sup>st</sup> floor lobby)*

	Parallel sessions, Lecture hall A1, Medical tomography	Parallel sessions, Lecture hall A3, Forward & inverse modeling from electrode measurements	Parallel sessions, Lecture hall A4, Material analysis
9:00–9:25	Eero Koponen	Fernando Galaz Prieto	Reda Elkhadrawy
9:25–9:50	Siiri Rautio	Joonas Lahtinen	Salla Latva-Äijö
9:50–10:15	Bjørn Jensen	Topi Kuutela	Jacques Cuenca

10:15–10:45 *Coffee (campus), Posters (on-site)*

Morning session 3, Lecture hall A1	
10:45–11:30	Lauri Mehtätalo: Model-based estimation of forest characteristics using remotely sensed data
11:30–12:00	Vincent Verhoeven: Quantifying TLS data and tree reconstruction uncertainty

12:00–13:00 *Lunch (restaurant)*

	Parallel sessions, Lecture hall A1, Remote sensing	Parallel sessions, Lecture hall A3, Manifolds II	Parallel sessions, Lecture hall A4, Parameter dependent PDEs
13:00–13:25	Sebastian Springer	Jinpeng Lu	Antti Autio
13:25–13:50	Janne Hakkarainen	Hjørdis Amanda Schlüter	Vesa Kaarnioja

13:50–14:20 *Coffee break (portrait foyer, 2<sup>nd</sup> floor), Posters (on-site)*

Afternoon session 3, Lecture hall A1	
14:20–14:50	Narayan P Subramaniyam: Causal coupling inference from multivariate time series based on ordinal partition transition networks
14:50–15:20	Alexandra Koulouri: Real-time ionospheric imaging of scintillation from limited data with parallel Kalman filters
15:20–15:50	Konstantinos Zygalakis: Bayesian inverse problems, prior modelling and algorithms for posterior sampling

15:50–16:00 *Conference closing (Lecture hall A1)*

## Day 1, Tuesday Dec 14th

**10:00 - 10:45 – Registration, 1st floor looby**

**Conference opening: Lecture Hall A1**

Session Chair: Pasi Raumonen

10:45 - 11:00 - Samuli Siltanen | University of Helsinki, Finland

**Morning session 1: Lecture Hall A1**

Session Chair: Sampsa Pursiainen

11:00 - 11:45 - Stephanie Lohrengel | University of Reims Champagne-Ardenne, France | Diffusive optical tomography in the neonatal head: a homogenized model for the cerebrospinal fluid layer

11:45 - 12:30 - Christelle Eyraud | Institut Fresnel, Aix-Marseille University, France | Microwave imaging from manufactured targets to small Solar bodies

**12:30 - 13:30 - Lunch (campus)**

**12:30 - 14:30 - Board meeting**

**Session 1: Asteroid numerical modelling | Lecture Hall A1**

Session Chair: Anne Virkki

13:30 - 13:55 - Liisa-Ida Sorsa | Tampere University, Finland | Full-Wave Radar Tomography of Complex, High-Contrast Targets: Imaging a Small Solar System Body

13:55 - 14:20 - Astrid Dufaure | Institut Fresnel, Aix-Marseille University, France | Internal imaging of asteroid analogues by electromagnetic method

14:20 - 14:45 - Yusuf Oluwatoki Yusuf | Tampere University, Finland | Full-Wave Radar Tomography: Application to Small Solar System Bodies in the Presence of Wavelength-induced Errors

**Session 2: Gaussian priors and Correction models | Lecture Hall A2a**

Session Chair: Tuomo Valkonen

13:30 - 13:55 - Hanne Kekkonen | Delft University of Technology, The Netherlands | Consistency of Bayesian inference for a parabolic inverse problem

13:55 - 14:20 - Andreas Hauptmann | University of Oulu, Finland | Learned operator correction in inverse problems

14:20 - 14:45 - Jenni Poimala | University of Oulu, Finland | Learned speed of sound correction for photoacoustic tomography

14:20 - 14:45 - Lauri Ylinen | University of Helsinki, Finland | Complex-Valued Artificial Neural Networks

**Session 3: Stroke classification | Lecture Hall A3**

Session Chair: Marko Vauhkonen

13:30 - 13:55 - Samuli Siltanen | University of Helsinki, Finland | A new view of electrical impedance tomography

13:55 - 14:20 - Fernando Moura | University of Helsinki, Finland | Anatomical atlas of the upper part of the human head for electroencephalography and bioimpedance applications

14:20 - 14:45 - Rashmi Murthy | University of Helsinki, Finland | Combing deep learning with the Complete Electrode Measurements of EIT to classify stroke

## **Session 4: Acoustic tomography | Lecture Hall A4**

Session Chair: Pasi Raumonen

13:30 - 13:55 - Tanja Tarvainen | University of Eastern Finland, Finland | Quantitative Photoacoustic Tomography in a Bayesian Framework

13:55 - 14:20 - Shiqi Ma | University of Jyväskylä, Finland | Fixed angle inverse scattering for sound speeds close to constant

14:20 - 14:45 - Niko Hänninen | University of Eastern Finland, Finland | Utilizing Monte Carlo method for light transport in quantitative photoacoustic tomography

**14:45 - 15:15 - Coffee break, Portrait Foyer, 2nd floor**

## **Afternoon session: Lecture hall A1**

Session Chair: Tapio Helin

15:15 - 16:00 - Matti Lassas | University of Helsinki, Finland | Inverse problems for a random walk on a finite graph

16:55 - 16:30 - Anne Virkki | University of Helsinki, Finland | Inverse modeling of near-Earth asteroids using ground-based planetary radar observations

16:30 - 17:00 - Abdu Mohammed Seid | Bahir Dar University, Ethiopia | The Spatiotemporal Dynamics of Water Hyacinth over Lake Tana of Ethiopia using Hidden Potts prior and Dynamic Linear Models

**17:00 - 17:30 - Snacks & Beverages**

## **Posters/E-posters**

Session Chair: Atena Rezaei

17:30 - 17:40 - Manu Holmberg | Finnish Meteorological Institute, Finland | Retrieving snow density from passive microwave measurements at L-band

17:40 - 17:50 - Arttu Arjas | University of Oulu, Finland | Neural Network Kalman filtering for 3D object tracking from linear array ultrasound data

17:50 - 18:00 - Katrine O. Bangsgaard | Technical University of Denmark, Denmark | Low-rank flat-field correction for artifact reduction in spectral CT

18:00 - 18:10 - Silja Westphal Christensen | University of Denmark, Denmark | Structural Gaussian Priors for Bayesian CT Reconstruction of Flexible Subsea Pipes

18:10 - 18:20 - Teemu Salminen | University of Eastern Finland, Finland | Application of finite element method to General Dynamic Equation of Aerosols - comparison with classical numerical approximations

18:20 - 18:30 - Antti Mikkonen | Finnish Meteorological Institute, Finland | Sensitivity of radiance simulations over snow for satellite-based remote sensing of carbon dioxide

18:30 - 18:40 - Kristoffer Linder-Steinlein | Technical University of Denmark, Denmark | Spectral analysis of the stochastic Helmholtz equation in the plane

18:40 - 18:50 - Sampsa Pursiainen | Tampere University, Finland | Numerical Modelling for the Juventas Radar (JuRa) investigation: Analysis of simulated full-wave scattering data for test fragments in time domain

**19:00 - 20:30 - Reception at Café & Aula Toivo (campus)**

**20:30 - Unofficial after-party at Restaurant Plevna**

### Day 2, Wednesday Dec 15<sup>th</sup>

**8:45 - 9:00 – Registration, 1st floor looby**

**Session 5: Bayesian Approach in EEG/MEG Source Imaging and Functional Brain Connectivity | Lecture Hall A1**

Session Chair: Andreas Hauptmann

9:00 - 9:25 - Alberto Sorrentino | Università degli Studi di Genova, Italy | Fully Bayesian approaches in M/EEG source imaging: an introduction

9:25 - 9:50 - Sara Sommariva | CNR-SPIN, Italy | Lasso-based estimation of the cross-power spectrum in linear inverse problems and its application to functional brain connectivity

9:50 - 10:15 - Alessandro Viani | Università degli Studi di Genova, Italy | Selection and averaging of hyperparameter comes for free with SMC samplers

**Session 6: Helsinki Deblur Challenge 2021 | Lecture Hall A2a**

Session Chair: Samuli Siltanen

9:00 - 9:25 - Ji Li | National University of Singapore, Singapore | Deep Learning Approach with Conditional Blur Level Information

9:25 - 9:50 - Daniel Pelt | LIACS, Leiden University, The Netherlands | Using mixed-scale dense CNNs, multiple scales, and simulated training data for the Helsinki Deblur Challenge 2021

9:50 - 10:15 - Theophil Trippe | Technical University of Berlin, Germany | Learning to Invert Defocus Blur: A Data-Driven Approach to the "Helsinki Deblur Challenge"

**Session 7: Harmonic operators | Lecture Hall A3**

Session Chair: Lassi Roininen

9:00 - 9:25 - Boya Liu | North Carolina State University, USA | Stability estimates in a partial data inverse boundary value problem for biharmonic operators at high frequencies

9:25 - 9:50 - Jaakko Kultima | University of Oulu, Finland | Inverse scattering for quasi-linear biharmonic operator in 2D

9:50 - 10:15 - Suman Sahoo | University of Jyväskylä, Finland | Unique determination of Anisotropic perturbations of a polyharmonic operator from partial boundary data

**Session 8: Stability and perturbations | Lecture Hall A4**

Session Chair: Lauri Oksanen

9:00 - 9:25 - Giovanni Covi | University of Heidelberg, Germany | Uniqueness for the fractional Calderon problem with quasilocal perturbations

9:25 - 9:50 - Pu-Zhao Kow | University of Jyväskylä, Finland | Optimality of increasing stability for an inverse boundary value problem

9:50 - 10:15 - Leyter Potenciano-Machado | University of Jyväskylä, Finland | An inverse problem for a semi-linear wave equation

**10:15 - 10:45 – Coffee break, Portrait Foyer, 2nd floor**

## **Morning session 2: Lecture Hall A1**

Session Chair: Joonas Ilmavirta

10:45 - 11:15 - Atena Rezaei | Tampere University, Finland | Reconstructing subcortical and cortical activity of Somatosensory via Inverse modeling techniques utilizing SEP datasets

11:15 - 12:00 - Carsten H. Wolters | University of Münster, Germany | New non-invasive multimodal neuroimaging and neurostimulation methods for improved diagnosis and therapy in refractory focal epilepsy

### **12:00 - 13:00 - Lunch (campus)**

Session Chair: Tanja Tarvainen

12:30 - 13:30 – Women in Inverse Problems (cabinet)

## **Session 9: FEM-Based Modeling & Inversion in EEG/MEG | Lecture Hall A1**

Session Chair: Pasi Raumonon

13:30 - 13:55 - Tim Erdbrügger | University of Münster, Germany | A Cut finite element method for the EEG forward problem

13:55 - 14:20 - Frank Neugebauer | Tampere University, Finland | Validating EEG, MEG and combined MEG and EEG beamforming for an estimation of the epileptogenic zone in two focal cortical dysplasia epilepsy patients

14:20 - 14:45 - Malte Höltershinken | University of Münster, Germany | Block Krylov Solvers for Transfer Matrix Computations in Bioelectromagnetism

## **Session 10: Electromagnetic waves data | Lecture Hall A2a**

Session Chair: Mikko Salo

13:30 - 13:55 - Marzieh Hosseini | University of Eastern Finland, Finland | Electrical capacitance tomography assisted control in a microwave drying process

13:55 - 14:20 - Snizhana Ross | University of Oulu, Finland | Hierarchical Deconvolution for Incoherent Scatter Radar Data

14:20 - 14:45 - Manuel Pena | Universidad Politécnica de Madrid, Spain | Testing the topological derivative against experimental data; the Institut Fresnel database

## **Session 11: Manifold I | Lecture Hall A3**

Session Chair: Nuutti Hyvonen

13:30 - 13:55 - Joonas Ilmavirta | University of Jyväskylä, Finland | Breaking cosmological conformal gauge with neutrinos

13:55 - 14:20 - Janne Nurminen | University of Jyväskylä, Finland | An inverse problem for the minimal surface equation

14:20 - 14:45 - Antti Kykkänen | University of Helsinki, Finland | Pestov identities and X-ray tomography on manifolds with non-smooth Riemannian metrics

### **Session 12: Tomography and resources | Lecture Hall A4**

Session Chair: Tanja Tarvainen

13:30 - 13:55 - Tommi Heikkilä | University of Helsinki, Finland | Sparse Dynamic Tomography Regularization with 4D Representation Systems

13:55 - 14:20 - Angele Niclas | Institut Camille Jordan, France | High sensibility imaging of defects in waveguides using near resonance frequencies

14:20 - 14:45 - Alexander Meaney | University of Helsinki, Finland | Experimental and Computational Resources for Computed Tomography Research at the University of Helsinki

**14:45 - 15:15 - Coffee break, Portrait Foyer, 2nd floor**

### **Afternoon session: Lecture hall A1**

Session Chair: Sampsa Pursiainen

15:15 - 16:00 - Francesca Pitolli | University of Roma 'La Sapienza', Italy | Randomized Sampling Methods for Linear Inverse Problems

16:55 - 16:30 - Sarah Hamilton | Marquette University, USA | Fast 3D CGO-Based Reconstruction for Absolute Electrical Impedance Tomography on Experimental Data from ACT5

16:30 - 17:00 - Annalisa Pascarella | IAC-CNR, Italy | An in-vivo comparison of source localization methods

**17:00 - 18:00 - FIPS meeting**

**19:00 - 21:00 - Dinner at Tampella restaurant**

## Day 3, Thursday Dec 16th

### Session 13: Medical tomography | Lecture Hall A1

Session Chair: Andreas Rupp

9:00 - 9:25 - Eero Koponen | University of Eastern Finland, Finland | Nonlinear estimation of potential flow in background-oriented schlieren imaging

9:25 - 9:50 – Siiri Rautio | University of Helsinki, Finland | Learning a microlocal prior for limited-angle tomography

9:50 - 10:15 - Bjorn Jensen | University of Helsinki, Finland | Sound speed uncertainty in Acousto-Electric Tomography

### Session 14: Forward & Inverse modeling from electrode measurements | Lecture Hall A2a

Session Chair: Joonas Ilmavirta

9:00 - 9:25 - Fernando Galaz Prieto | Tampere University, Finland | L1 vs. L2 norm fitting in optimizing focal multi-channel tES stimulation: linear programming vs. weighted and re-weighted least-squares

9:25 - 9:50 - Joonas Lahtinen | Tampere University, Finland |  $\ell_1$ -Reweight Hierarchical Bayesian Method for EEG Source Localization via Randomized Multiresolution Scanning

9:50 - 10:15 - Topi Kuutela | Aalto University, Finland | Contact adapting electrode model for electrical impedance tomography

### Session 15: Material analysis | Lecture Hall A3

Session Chair: Sampsa Pursiainen

9:00 - 9:25 - Reda Elkhadrawy | Tampere University, Finland | Inverse Approach for Identifying Intrinsic Electromagnetic Material Parameters of Ferrite Cores

9:25 - 9:50 - Salla Latva-äijö | University of Helsinki, Finland | Material-separating regularizer for multi-energy X-ray tomography

9:50 - 10:15 - Jacques Cuenca | Siemens Industry Software, Belgium | Deterministic and statistical characterisation of poroelastic media from sound absorption measurements

**10:15 - 10:45 – Coffee break, Portrait Foyer, 2nd floor**

### Morning session 3: Lecture Hall A1

Session Chair: Pasi Raumonon

10:45 - 11:15 - Lauri Mehtätalo | Natural Resources Institute Finland (Luke), Finland Model-based estimation of forest characteristics using remotely sensed data

11:15 - 12:00 - Vincent Verhoeven | Tampere University, Finland | Quantifying TLS data and tree reconstruction uncertainty

**12:00 - 13:00 - Lunch (campus)**

### Session 16: Remote sensing | Lecture Hall A1

Session Chair: Pasi Raumonon

13:00 - 13:25 - Sebastian Springer | University of Oulu, Finland | Sawing Optimization, a Tandem Forest values project



13:25 - 13:50 - Janne Hakkarainen | Finnish Meteorological Institute, Finland | Analyzing nitrogen oxides to carbon dioxide emission ratios from space: A case study of Matimba Power Station in South Africa

### **Session 17: Manifold II | Lecture Hall A2a**

Session Chair: Antti Hannukainen

13:30 - 13:25 - Jinpeng Lu | University of Helsinki, Finland | Stability of the Gel'fand's inverse boundary spectral problem via the unique continuation

13:25 - 13:50 - Hjørdis Amanda Schlüter | Technical University of Denmark, Denmark | Reconstructing anisotropic conductivities on 2D Riemannian manifolds from power densities

### **Session 18: Parameter dependent PDEs | Lecture Hall A3**

Session Chair: Sampsa Pursiainen

13:30 - 13:25 - Antti Autio | Aalto University, Finland | A compound Krylov subspace method for parametric linear systems

13:25 - 13:50 - Vesa Kaarnioja | LUT University, Finland | Quasi-Monte Carlo methods for optimal control problems subject to time-dependent PDE constraints under uncertainty

### **13:50 - 14:20 - Coffee break, Portrait Foyer, 2nd floor**

Afternoon session: Lecture hall A1

Session Chair: Joonas Ilmavirta

14:20 - 14:50 - Narayan P Subramaniam | Tampere University, Finland | Causal coupling inference from multivariate time series based on ordinal partition transition networks

14:50 - 15:20 - Alexandra Koulouri | Tampere University, Finland | Real-time Ionospheric Imaging of Scintillation from Limited Data with Parallel Kalman Filters

15:20 - 15:50 - Konstantinos Zygalakis | The University of Edinburgh, Scotland, United Kingdom | Bayesian inverse problems, prior modelling and algorithms for posterior sampling

15:50 - 16:00 - Conference closing, Lecture hall A1

Session Chair: Joonas Ilmavirta

## Abstracts chronologically

## Diffusive optical tomography in the neonatal head: a homogenized model for the cerebrospinal fluid layer

Stephanie Lohrengel<sup>1</sup>, Mahdi Mahmoudzadeh<sup>2</sup>, Farah Oumri<sup>1</sup>, Stéphanie Salmon<sup>1</sup>, Fabrice Wallois<sup>2</sup>

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<sup>2</sup> GRAMFC UMR-S 1105 INSERM, Amiens University Hospital CHU Sud, rue René Laennec, F-80054 Amiens Cedex 1, France.

Diffusive optical tomography (DOT) is a noninvasive imaging modality based on the relative transparency of biological tissues to light in the near-infrared (NIR) range. Near infrared spectroscopy (NIRS) is widely used for monitoring critical ill newborns since the technique is available at the bed side of the patient. It is also a complementary modality to electroencephalography (EEG) for the detection and localization of epileptic seizures. NIRS and its imaging counterpart DOT is based on the principle that hemodynamic changes in the concentration of the dominant chromophores oxyhemoglobin (HbO) and deoxyhemoglobin (HbR) induce changes in the absorption and scattering properties of the tissues.

From a mathematical point of view, the radiative transfer equation (RTE) is an appropriate model for light propagation in biological tissues. Due to the high computational cost of the numerical resolution of the RTE, the diffusion approximation is widely used. However, it is known that the classical diffusion approximation is at the limit of validity in the cerebrospinal fluid (CSF) [1].

In this communication, we present a new model for optical tomography that takes into account the fine substructure of CSF and is obtained by the diffusion approximation of the homogenized RTE. We address the validity of the new model from a physiological and mathematical point of view and discuss its performance in view of sensitivity to perturbations in the brain layer [2].

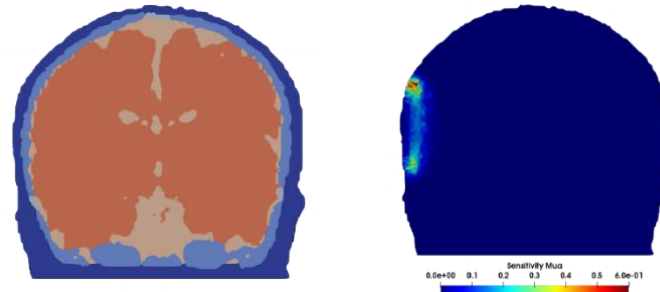


Figure 1: Coronal cross section of the neonatal head model (left). Sensitivity with respect to a source/detector couple in the temporal lobe (right).

- [1] E. Okada, D.T. Delpy. Near-infrared light propagation in an adult head model. I. Modeling of low-level scattering in the cerebrospinal fluid layer. *Appl. Optics*. 2003;42(16), pp. 2906–2914.
- [2] S. Lohrengel, M. Mahmoudzadeh, F. Oumri, S. Salmon, F. Wallois. A homogenized cerebrospinal fluid model for diffuse optical tomography in the neonatal head, *Int. J. Num. Meth. Biomed. Engng*, doi.org/10.1002/cnm.3538, 2021.

Acknowledgements: This work has received funding from the French National Research Agency (grant ANR-15-CE23 0009, MAIA project)

## Microwave imaging from manufactured targets to small Solar bodies

Christelle Eyraud<sup>1</sup>, Jean-Michel Geffrin<sup>1</sup>, Amélie Litman<sup>1</sup>, Astrid Dufaure<sup>1</sup>

<sup>1</sup> Aix Marseille Univ., CNRS, Centrale Marseille, Institut Fresnel, Marseille, France.

Electromagnetic wave probing is an interesting tool to obtain characteristics of unknown targets (position, shape, size, complex permittivity) in a non destructive way. Indeed, the physical features can be retrieved from the measurements of the target's scattered field thanks to the resolution of an inverse problem [1]. In practice, there are some difficulties as the scattered field can rarely be measured in the near field, it is a priori measured in the far field and only on a part of the surface surrounding the target, due to time limitation or other constraints. Moreover, all the polarization cases of the scattered field are rarely measured. The data are therefore necessarily truncated. Finally, the measurements are also ineluctably affected by noise

In this presentation, after discussing inverse techniques for 3D target imaging and illustrating this with examples of reconstructed targets obtained from electromagnetic measurements in the laboratory, we will focus on imaging the internal structure of small solar system bodies (asteroids and comets). Indeed, several missions including radar are currently planned to probe the internal structure of comets and asteroids [2], [3].

- [1] A. Litman, L. Crocco, Testing inversion algorithms against experimental data: 3D targets Guest editors' Introduction, *Inverse Problems*, 25, 2009.
- [2] W Kofman et al. Properties of the 67P/Churyumov-Gerasimenko interior revealed by CONSERT radar. *Science*, 349(6247), 2015.
- [3] A. Hérique et al., Direct observations of asteroid interior and regolith structure: Science measurement requirements, *Advances in Space Research*, 62, 2018.

# Full-Wave Radar Tomography of Complex, High-Contrast Targets: Imaging the Interior Structure of a Small Solar System Body

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<sup>1</sup> Computing Sciences, Faculty of Information Technology and Communication Sciences, Tampere University, P.O. Box 692, FI-33101 Tampere, Finland.

Imaging the interior structure of an asteroid, a Small Solar System Body (SSSB), presents a challenging mathematical and computational, ill-posed inverse problem. This presentation is based on my doctoral dissertation with the same title, examined at Tampere University in the end of October 2021 [1]. In this work, the problem is investigated by applying the finite element time-domain (FETD) method to simulate the full wavefield in the forward problem.

The problem is investigated from three different angles: (1) feasibility of carrying out inversion of the data to detect deep interior details in an asteroid model [2], (2) method development of expanding from the linearized problem to the non-linear one by formulating the higher-order Born Approximation of the wavefield [3], and (3) Comparing simulated data and results to microwave radar measurements on an analogue created based on the numerical target [4,5,6].

The results show that tomographic imaging of a realistic rubble-pile asteroid target is feasible, and that interior details can be detected with a low-frequency radar. The future work entails development of the modelling methods to investigate inversion of tomographic data with higher frequencies such as those specified for the Juventas Radar in European Space Agency's Hera mission.

- [1] Sorsa L.-I. Full-Wave Radar Tomography of Complex, High-Contrast Targets: Imaging the Interior Structure of a Small Solar System Body. Doctoral Dissertation. Tampere University (2021). <http://urn.fi/URN:ISBN:978-952-03-2127-7>.
- [2] Sorsa L.-I., Takala M., Bambach P., Deller J., Vilenius E., Pursiainen S. Bistatic full-wave radar tomography detects deep interior voids, cracks and boulders in a rubble-pile asteroid model. *The Astrophysical Journal*, 872.1 (2019), 44. DOI: 10.3847/1538-4357/aafba2.
- [3] Sorsa L.-I., Takala M., Eyraud C., Pursiainen S. A Time-Domain Multigrid Solver with Higher- Order Born Approximation for Full-Wave Radar Tomography of a Complex-Shaped Target. *IEEE Transactions on Computational Imaging*, 6 (2020), pp. 579-590. DOI: 10.1109/TCI.2020.2964252.
- [4] Eyraud C., Sorsa L.-I., Geffrin J.-M., Takala M., Henry G., Pursiainen S. Full Wavefield Simulation vs. Measurement of Microwave Scattering by a Complex 3D-Printed Asteroid Analogue. *Astronomy & Astrophysics*, 643 (2020), pp. A68. DOI: 10.1051/0004-6361/202038510.
- [5] Sorsa L.-I., Eyraud C., Hérique A., Takala M., Pursiainen S., Geffrin J.-M. Complex-Structured 3D-Printed Wireframes as Asteroid Analogues for Tomographic Microwave Radar Measurements. *Materials & Design*, 198 (2021) , pp. 109364. DOI: 10.1016/j.matdes.2020.109364.
- [6] Sorsa L.-I., Pursiainen S., Eyraud C. Analysis of full microwave propagation and backpropagation for a complex asteroid analogue via single-point quasi-monostatic data. *Astronomy and Astrophysics*, 645 (2021), pp. A73. DOI: 10.1051/0004-6361/202039380.
- [7] P. Michel, M. Kueppers, H. Sierks, I. Carnelli, A. F. Cheng, K. Mellab, M. Granvik, A. Kestilä, T. Kohout, K. Muinonen et al. European component of the AIDA mission to a binary asteroid: Characterization and interpretation of the impact of the DART mission. *Advances in Space Research* 62.8 (2018), 2261–2272. DOI: 10.1016/j.asr.2017.12.020.

Acknowledgements: This work was supported by Academy of Finland Centre of Excellence in Inverse Modelling and Imaging 2018-2025 (project number 312341), Emil Aaltonen Foundation, and Academy of Finland ICT 2023 programme (project number 336151).

## Internal imaging of asteroid analogues by electromagnetic method

A. Dufaure<sup>1,2</sup>, Y. Yusuf<sup>1,2</sup>, J-M. Geffrin<sup>1</sup>, S. Pursiainen<sup>2</sup>, L-I. Sorsa<sup>2</sup>, A. Litman<sup>1</sup>, C. Eyraud<sup>1</sup>

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In order to better understand the formation of our solar system and, in our case, the small bodies of the solar system (asteroids, comets), it is necessary to develop new remote sensing and imaging methods. We seek to image the internal structure of comets and asteroids using the radar technique that is widely applied in various other fields (geophysics, medical, etc.). This was the case with the CONSERT (COMet Nucleus Sounding Experiment by Radiowave Transmission) radar on board the Rosetta space mission to comet 67P/Churyumov-Gerasimenko. CONSERT has led to numerous scientific discoveries [1], [2] and has inspired new space missions such as the Hera mission (rendezvous 2026) which will send the Juventas Radar to the binary asteroid 65803 Didymos.

In order to anticipate the exploitation of the results, adequate imaging techniques must be implemented and tested on similar objects with laboratory measurements. In this study we aim to provide a fast and efficient method to image the external shape and interior of small solar bodies. For this purpose, measurements were performed on an analog [3], [4]. The use of imaging methods allows us to image the external structure of the analogue as well as the interior.

- [1] W. Kofman et al., Properties of the 67P/Churyumov-Gerasimenko interior revealed by CONSERT radar, vol. 349, issue 6247, *Cometary Sciences*, 2015
- [2] A. Hérique et al., Cosmochemical implications of CONSERT permittivity characterization of 67P/CG, vol. 462, *Monthly Notices of the Royal Astronomical Society*, 2016, pp. S516-S532.
- [3] L-I. Sorsa et al., Complex-structured 3D-printed wireframes as asteroid analogues for tomographic microwave radar measurements, vol. 198, *Materials & Design*, 2021, pp.109364.
- [4] A. Fujiwara et al., The rubble-pile asteroid Itokawa as observed by Hayabusa, vol. 321, N. 5778, *Science*, 2006, pp. 1330-1334

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Acknowledgements: The authors acknowledge the opportunity provided by the Centre Commun de Ressources en Microonde to use its fully equipped anechoic chamber and the Mesocentre of Aix-Marseille University.

## Full-Wave Radar Tomography: an Application to Small Solar System Bodies in the Presence of Wavelength-induced Errors

Yusuf Oluwatoki Yusuf<sup>1,2</sup>, Astrid Dufaure<sup>2</sup>, Liisa-Ida Sorsa<sup>1</sup>, Christelle Eyraud<sup>2</sup>, Sampsa Pursiainen<sup>1</sup>

<sup>1</sup> Computing Sciences, Tampere University (TAU), P.O. Box 692, 33101, Tampere, Finland

<sup>2</sup> Aix-Marseille Univ., CNRS, Centrale Marseille, Institut Fresnel, Marseille, France

There are over 27,313 known NEAs, i.e., within  $1.945 \times 10^8$  km from the sun and only  $4.488 \times 10^7$  km from Earth, of which 1,254 are on the risk list of ESA's Near Earth Object Coordination Centre (NEOCC). Hence, advancing the knowledge of mineral composition, interior structure, and potential threats of planetary bodies is very important which has been the goal of space explorations over years. The first space mission to a small Solar System Body (sSSB), aimed at performing Radar Tomography (RT) was Comet Nucleus Sounding Experiment by Radio Transmission (CONSERT) by European Space Agency's (ESA's) Rosetta mission in 2004. ESA will continue RT exploration as a part of the HERA mission [2, 3], the European component of AIDA (Asteroid Impact and Deflection Assessment), whose Juventas Radar (JuRa) [1] carried by a CubeSat with the same name (Juventas CubeSat), will perform RT measurements with the asteroid moon of the binary asteroid Didymos as its target. Reconstructing an SSSB's interior permittivity distribution (structure) via RT is an ill-posed inverse problem and utilising the full-waveform approach, i.e., maintaining the complete wavefield information of the forward simulation, is essential with respect to the reconstruction quality.

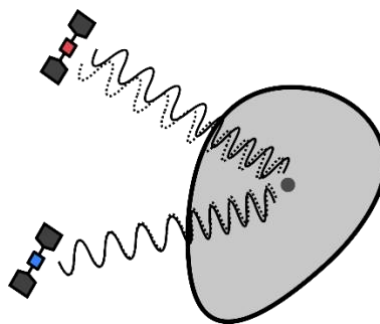


Figure 1: Modelling errors as a result of phase discrepancy leading to large random fluctuations in the demodulated baseband data.

We aim to investigate the interior structure of a two-dimensional test object via numerically simulated full-wave time domain radar tomography with the presence of carrier wave induced uncertainty, hence formulating a statistical model to marginalise such uncertainty.

- [1] Alain Herique, Dirk Plettemeier, Hannah Goldberg, and Wlodek Kofman. Jura: the juventas radar on hera to fathom didymoon. In *European Planetary Science Congress*, pages EPSC2020–595, 2020.
- [2] Alain Herique, Dirk Plettemeier, Wlodek W Kofman, Yves Rogez, and Hannah Goldberg. Direct observations of didymos' regolith and internal structure with lfr radar on juventas cubesat for esa hera mission. In *AGU Fall Meeting Abstracts*, volume 2019, pages NH51C–0789, 2019.
- [3] Patrick Michel, Michael Kuppers, and Ian Carnelli. The hera mission: European component of the esa-nasa aida mission to a binary asteroid. *42nd COSPAR Scientific Assembly*, 42:B1–1, 2018.

Acknowledgements: This work was supported by the Academy of Finland Centre of Excellence in Inverse Modelling and Imaging, 2018-2025

## Consistency of Bayesian inference for a parabolic inverse problem

Hanne Kekkonen<sup>1</sup>

<sup>1</sup> Delft Institute for Applied Mathematics, Delft University of Technology, Netherlands.

Bayesian methods for inverse problems have become increasingly popular in applied mathematics in the last decades but the theoretical understanding of the statistical performance of these methods for non-linear inverse problems is still developing. In this talk I will establish posterior contraction rates for a non-linear parabolic inverse problem with rescaled Gaussian process priors. More precisely, the inverse problem of discovering the absorption term  $f > 0$  in a heat equation, with given boundary and initial value functions, from  $N$  discrete noisy point evaluations of the forward solution is considered. I will also show that the optimal minimax rate can be achieved with truncated Gaussian priors.

[1] Hanne Kekkonen. Consistency of Bayesian inference with Gaussian process priors for a parabolic inverse problem. *arXiv preprint*, arXiv:2103.13213, 2021.



## Learned operator correction in inverse problems

Andreas Hauptmann

University of Oulu

Iterative model-based reconstruction approaches for high-dimensional problems with non-trivial forward operators can be highly time consuming. Thus, it is desirable to employ model reduction techniques to speed-up reconstructions in variational approaches as well as to enable training of learned model-based techniques. Nevertheless, reduced or approximate models can lead to a degradation of reconstruction quality and need to be accounted for. For this purpose, we discuss in this talk the possibility of learning a data-driven explicit model correction for inverse problems and whether such a model correction can be used within a variational framework to obtain regularized reconstructions. We will discuss the conceptual difficulty of learning such a forward model correction and derive conditions under which solutions to the variational problem with a learned correction converge to solutions obtained with the accurate model.

## Learned speed of sound correction for photoacoustic tomography

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<sup>1</sup> Research Unit of Mathematical Sciences, University of Oulu, P.O. Box 8000, FI-90014 Oulu, Finland.

<sup>2</sup> Department of Computer Science, University College London, London WC1E 6BT, United Kingdom.

Photoacoustic tomography is a hybrid imaging modality that combines optical contrast with ultrasonic detection and it has shown the potential in a variety of biomedical applications. Real-time applications in three-dimensional photoacoustic tomography are relying on fast reconstruction algorithms, instead of utilising full wave solvers. Unfortunately, the majority of available fast solvers assume homogenous speed of sound distributions. Additionally, reconstruction quality depends on the correct choice for a constant speed of sound in the imaged target. In practical experiments, accurate knowledge of the speed of sound of the imaged target is commonly not available. To overcome these limitations, we propose to learn a data-driven correction in k-space to compensate for unknown or heterogeneous speed of sound distributions, when fast FFT based methods are used to compute reconstructions. We evaluate the potential to include such a correction into the reconstruction algorithm in combination with a learned post-processing and compare to post-processing alone. The feasibility of the approach is studied with simulated data and human in vivo measurements.

## Complex-Valued Artificial Neural Networks

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<sup>1</sup> Department of Mathematics and Statistics, University of Helsinki, P.O. Box 64, FI-00014 University of Helsinki, Finland.

Artificial neural networks are usually considered to be real-valued, i.e., the numbers that propagate through the network are real numbers. In many cases, however, the natural domain for the inputs of the network is the space of complex numbers. This is the case for any data represented in the frequency domain, for example. In this talk, we review complex-valued artificial neural networks from the point of view of inverse problems, and discuss their approximation capabilities, namely, for which activation functions  $\sigma: U \rightarrow \mathbb{C}$  (where  $U$  is an open subset of  $\mathbb{C}$ ) the associated networks can uniformly approximate an arbitrary continuous function. This is joint work with Matti Lassas, Tuomo von Lerber, and Franko Küppers.

## A new view of electrical impedance tomography

Samuli Siltanen

Stroke is a leading cause of death all around the world. There are two main types of stroke: ischemic (blood clot preventing blood flow to a part of the brain) and hemorrhagic (bleeding in the brain). The symptoms are the same, but treatments very different. A portable “stroke classifier” would be a life-saving equipment to have in ambulances, but so far it does not exist.

Electrical Impedance Tomography (EIT) is a promising and harmless imaging method for stroke classification. In EIT one attempts to recover the electric conductivity inside a domain from electric boundary measurements. This is a nonlinear and ill-posed inverse problem. The so-called Complex Geometric Optics (CGO) solutions have proven to be a useful theoretical and computational tools for EIT.

A new property of CGO solutions is presented, showing that a one-dimensional Fourier transform in the spectral variable provides a connection to parallel-beam Xray tomography of the conductivity. One of the consequences of this “nonlinear Fourier slice theorem” is a novel capability to recover inclusions within inclusions in EIT. Furthermore, noise in EIT data shows up as blur in the virtual sinogram, reframing almost all ill-posedness of EIT into that of deconvolution.

In practical imaging, measurement noise causes strong blurring in the recovered profile functions. However, machine learning (ML) algorithms can be combined with the nonlinear PDE techniques to overcome the problem. It turns out that simulated strokes are classified by ML much more accurately from nonlinear Fourier features than from the unprocessed measurements.

# Anatomical atlas of the upper part of the human head for electroencephalography and bioimpedance applications

Fernando Moura<sup>1,2</sup>, Roberto Beraldo<sup>1</sup>, Leonardo Ferreira<sup>1</sup>, Samuli Siltanen<sup>2</sup>

<sup>1</sup> Engineering, modelling and Applied Social Sciences Center, Federal University of ABC, Santo André, Brazil.

<sup>2</sup> Department of Mathematics and Statistics, University of Helsinki, Helsinki, Finland

Volume conductor problems in cerebral electrophysiology and bioimpedance do not have analytical solutions for nontrivial geometries and require a 3D model of the head and its electrical properties for solving the associated PDEs numerically. Ideally, the model should be made with patient-specific information. In clinical practice, this is not always the case and an average head model is often used. Also, the electrical properties of the tissues might not be completely known due to natural variability. Anatomical atlases are important tools for in silico studies on cerebral circulation and electrophysiology that require statistically consistent data, e.g., machine learning, sensitivity analyses, and as a benchmark to test inverse problem solvers.

The objective of this work is to develop a 4D (3D+T) statistical anatomical atlas of the electrical properties of the upper part of the human head for cerebral electrophysiology and bioimpedance applications [1]. The atlas was constructed based on 3D MRI of 107 human individuals and comprises the electrical properties of the main internal structures and can be adjusted for specific electrical frequencies. T1w+T2w MRI images were used to segment the main structures of the head while angiography MRI was used to segment the main arteries. The proposed atlas also comprises a time-varying model of arterial brain circulation, based on the solution of the Navier-Stokes equation in the main arteries and their vascular territories. The source code of the atlas and solver are freely available to download [2].

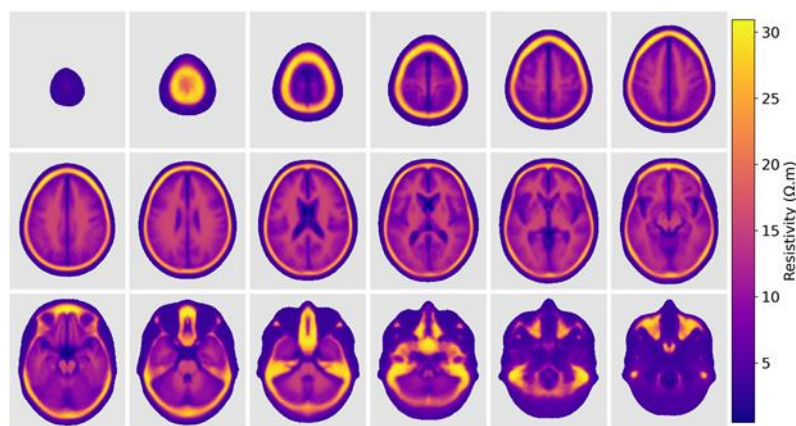


Figure 1: Transversal slices of the static component of the atlas (average, resistivity) at 1kHz.

[1] Moura et al 2021 *Physiol. Meas.* <https://doi.org/10.1088/1361-6579/ac3218>

[2] <https://github.com/fsmMLK/openSAHE>, <https://doi.org/10.5281/zenodo.5567086>

Acknowledgements: The work was funded in part by the Jane and Aatos Erkkö Foundation, the Academy of Finland (Centre of Excellence in Inverse Modelling and Imaging, decision number 312339), the São Paulo Research Foundation--FAPESP (Process numbers: 2019/09154-7 and 2017/18378-0) and Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) -- Finance Code 001.

## Combing deep learning with the Complete Electrode Measurements of EIT to classify stroke

Rashimi Murthy  
University of Helsinki

Electrical impedance tomography (EIT) is an imaging method based on probing an unknown conductive body with electrical currents. Voltages resulting from the current feeds are measured at the surface, and the conductivity distribution inside is reconstructed. This is a promising technique in medical imaging as various organs and tissues have different conductivities. The motivation of this talk arises from classifying the two different kinds of strokes in the brain, ischemic or haemorrhagic. Typical EIT images are not optimal for stroke-EIT because of blurred images. In this talk we present a neural network approach to classify the stroke using the EIT boundary measurements. Here, we first approximate the idealised boundary condition, that is Dirichletto-Neumann (DN) map, from the practical boundary electrode measurements of Complete Electrode Model (CEM). We then use this approximation of idealised DN map to extract robust features called Virtual Hybrid Edge Detection (VHED) functions that have a geometric interpretation and whose computation from EIT data does not involve calculating a full image of the conductivity. We report the measures of accuracy for the stroke prediction using VHED functions on datasets that differ from the training data used for the training of neural network.

## Quantitative Photoacoustic Tomography in a Bayesian Framework

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Photoacoustic tomography is a coupled physics imaging modality which combines optical contrast and high spatial resolution of ultrasound. In the inverse problem of photoacoustic tomography, one aims at estimating target optical parameters from measured photoacoustic waves generated by absorption of an externally introduced light pulse.

We approach the inverse problem of quantitative photoacoustic tomography in the framework of Bayesian inverse problems. Posterior distribution and its point estimates are examined also in situations in which the forward model contains uncertainties. Modelling of errors and their impact on the posterior distribution are investigated.

# Fixed angle inverse scattering for sound speeds close to constant

Shiqi Ma<sup>1</sup>, Leyter Potenciano-Machado<sup>1</sup>, and Mikko Salo<sup>1</sup>

<sup>1</sup>Department of Mathematics and Statistics, University of Jyväskylä, Jyväskylä, Finland

We study the fixed angle inverse scattering problem of determining a sound speed from scattering measurements corresponding to a single incident wave.

$$\begin{cases} (\eta(x) \partial_t^2 - \Delta)U = 0 & \text{in } \mathbb{R}^{n+1}, \\ U|_{\{t < -1\}} = \delta(t - x \cdot \theta). \end{cases}$$

Here  $\eta$  is supported in a bounded domain  $\Omega$ . The main result shows that a sound speed close to constant can be stably determined by just one measurement. Denoting the map from the sound speed  $\eta$  to the boundary data  $U$  as  $\mathcal{A}$ ,  $\mathcal{A} : \eta \mapsto U|_{(0,T) \times \Omega}$ , we establish the following Hölder stability,

$$\|\eta - 1\| \leq C \|\mathcal{A}(\eta) - \mathcal{A}(1)\|^\mu.$$

Our method is based on studying the linearized problem, which turns out to be related to the acoustic problem in photoacoustic imaging. We adapt the modified time-reversal method from [P. Stefanov and G. Uhlmann, *Thermoacoustic tomography with variable sound speed*, *Inverse Problems* **25** (2009), 075011] to solve the linearized problem in a stable way, and use this to give a local uniqueness result for the nonlinear inverse problem.

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Acknowledgements: S. M., L. P-M. and M. S. were supported by the Academy of Finland (Finnish Centre of Excellence in Inverse Modelling and Imaging, grant numbers 312121 and 309963), and M.S. was also supported by the European Research Council under Horizon 2020 (ERC CoG 770924).



## Utilizing Monte Carlo method for light transport in quantitative photoacoustic tomography

Niko Hänninen<sup>1</sup>, Aki Pulkkinen<sup>1</sup>, Simon Arridge<sup>2</sup>, Tanja Tarvainen<sup>1, 2</sup>

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<sup>2</sup> Department of Computer Science, University College London, Gower Street, WC1E 6BT, UK.

In photoacoustic tomography (PAT), images of an initial pressure distribution generated by absorption of an externally introduced light pulse are estimated [1]. The method combines unique contrast of optical absorption and high resolution of ultrasound. In quantitative photoacoustic tomography (QPAT), aim is to estimate concentrations of light absorbing molecules, chromophores, from photoacoustic images [2].

Image reconstruction problem of QPAT is an ill-posed inverse problem, which requires modeling the light transport inside the domain. A widely accepted method for light propagation in scattering medium, such as biological tissue, is Monte Carlo (MC) method for light transport. It is based on simulating and summing stochastic paths for photon trajectories in an absorbing and scattering medium. Simulating a large number of photons enables modeling light transport accurately, but on the other hand, fast and efficient reconstruction methods are crucial in practical applications. Thus, optimizing the number of simulated photon packets used in model based inversion is necessary to obtain estimates in a reasonable amount of time while concurrently simulating light propagation accurately.

In this work, the optical inverse problem of QPAT, i.e. estimation of optical parameters from the absorbed energy density, is approached in a Bayesian framework [3, 4]. Light transport is modeled using the MC method implemented in ValoMC software [5]. *Maximum a posteriori* (MAP) estimates are computed using a Gauss-Newton method in a stochastic optimization setting. We optimize the computation time of the method by adapting the number of photon packets used by the forward model in each Gauss-Newton iteration step. The photon packet count adaptation strategy is controlled by a so-called norm test set for expected relative error of the Gauss-Newton iteration direction [6]. The adaptation strategy is studied using numerical simulations.

- [1] Paul Beard. Biomedical photoacoustic imaging. *Interface Focus*, 1:602-631, 2011.
- [2] Ben Cox, Jan Laufer, Simon Arridge and Paul Beard. Quantitative spectroscopic photoacoustic imaging: a review. *J. Biomed. Opt.*, 17(6): 061202, 2012.
- [3] Tanja Tarvainen, Aki Pulkkinen, Ben Cox, Jari Kaipio and Simon Arridge. Bayesian image reconstruction in quantitative photoacoustic tomography. *IEEE Trans. Med. Imag.*, 32:2287-2298, 2013.
- [4] Alekski Leino, Tuomas Lunttila, Meghdoot Mozumder, Aki Pulkkinen and Tanja Tarvainen. Perturbation Monte Carlo method for quantitative photoacoustic tomography. *IEEE Trans. Med. Imag.*, 39(10):2985-2995, 2020.
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- [6] Callum Macdonald, Simon Arridge and Samuel Powell. Efficient inversion strategies for estimating optical properties with Monte Carlo radiative transport models. *J. Biomed. Opt.*, 25(8):085002, 2021

## Inverse problems for a random walk on a finite graph

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We study the inverse problem of determining a finite weighted graph  $(X, E)$  from the source-to-solution map on a vertex subset  $B \subset X$  for heat equations on graphs, where the time variable can be either discrete or continuous. We prove that this problem is equivalent to the discrete version of the inverse interior spectral problem, provided that there does not exist a nonzero eigenfunction of the weighted graph Laplacian vanishing identically on  $B$ . In particular, we consider inverse problems for discrete-time random walks on finite graphs. We show that under the Two-Points Condition, the graph structure and the transition matrix of the random walk can be uniquely recovered from the distributions of the first passing times on  $B$ , or from the observation on  $B$  of one realization of the random walk.

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## Inverse modeling of near-Earth asteroids using ground-based planetary radar observations

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More than 1140 asteroids have been observed using ground-based planetary radar systems at the Arecibo Observatory and Goldstone Deep Space Communications Complex. For many NEAs, Arecibo and Goldstone radar systems have been able to obtain delay-Doppler images at resolutions as fine as 7.5 m and 3.75 m, respectively. These images, in addition to the continuous wave (CW) measurements obtained for every successfully observed asteroid, provide us with insight to the physical properties of asteroids that can only be obtained using radar due to the high-precision ranging capabilities and long wavelengths probing the sub-surface. To derive the shape (e.g., Fig. 1, [1]) or the physical properties (such as the electric permittivity, mass density, and rubble size-frequency distribution) of the surfaces of asteroids from the radar images and CW measurements, we use inverse modeling among other techniques. In this presentation, I will discuss the different numerical inverse modeling methods that are being utilized and some that we are planning to develop to characterize asteroids using ground-based planetary radar observations in contrast to using only optical observations.

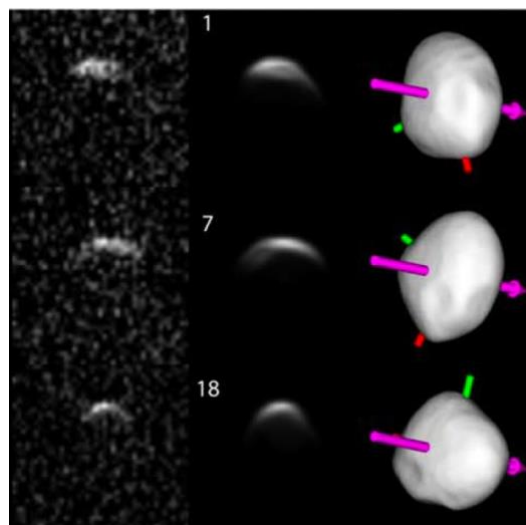


Figure 1: Example of inverse shape modeling using Arecibo radar observations of (16) Psyche [1].

[1] Michael Shepard et al. Asteroid 16 Psyche: Shape, Features, and Global Map. *Planetary Science Journal*, 2(125):16pp, 2021.

# The Spatiotemporal Dynamics of Water Hyacinth over Lake Tana of Ethiopia using Hidden Potts prior and Dynamic Linear Models

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The invasion of water hyacinth over lake Tana of Ethiopia, one of the largest lake in Africa and the source of the Blue Nile, has caused significant problems of biodiversity and resulting in high attentions and costs for eradicating this invasive aquatic weed. The objective of this study is therefore to investigate the spatiotemporal dynamics of the hyacinth over the lake and to identify the status of hyacinth infestation by classifying remotely sensed imageries of the lake. A time-series of multisensor high resolution imagery obtained from Landsat-8/OLI during the period 2013--2020 are analyzed to study the temporal, seasonal and spatial variability of the invasion that started in 2011 in lake Tana. We used a Bayesian model based image classification with Hidden Potts prior assumption to investigate the spatiotemporal distribution of the hyacinth over the lake during the study period. We also analyzed its seasonal and trend dynamics using a Dynamic Linear Model.



Figure 1: Study Area: Lake Tana, Ethiopia.

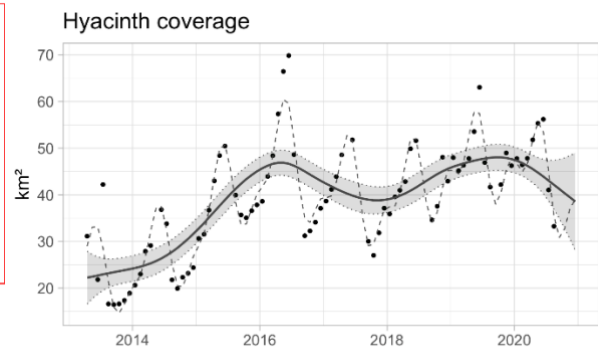


Figure 2: Seasonality and Trend Analysis of hyacinth coverage over lake Tana using DLM.

- [1] Dogliotti, A. I., Gossn, J. I., Vanhellemont, Q., and Ruddick, K. G. (2018). Detecting and quantifying a massive invasion of floating aquatic plants in the Río de la Plata turbid waters using high spatial resolution ocean color imagery. *Remote Sensing*, 10(7).
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- [4] Moores, M. T., Pettitt, A. N., and Mengersen, K. L. (2020b). Bayesian computation with intractable likelihoods. *Case Studies in Applied Bayesian Data Science*, pages 137–151.

## Retrieving snow density from passive microwave measurements at L-band.

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Snow cover and snow properties are important environmental parameters that are strongly linked to climate change, the amount of available freshwater, as well as control hydropower. The remote sensing of snow cover and its properties form a wide range of study and are of interest for many fields. While the passive microwave measurements from frequencies higher than 18GHz are traditionally used for the remote sensing of the snow cover, the passive microwave measurements from lower frequencies have not been fully utilized. Recent studies [1, 2] have shown that the passive microwave measurements from L-band (1 to 3 GHz) are sensitive to snow mass density and have demonstrated the possibility of retrieving the snow mass density from the measurements. Proposed retrieval methods employ a physical forward model simulating microwave emissions from snow covered ground. Parameters of interest are retrieved from observations using cost function minimization between model predictions and observations. This poster aims to present the problem of retrieving snow mass density from L-band passive microwave measurements, its motivation and background, as well as on going work on the uncertainty quantification of the retrievals by using Bayesian approach to the retrieval problem. The measurements used in the work are from ELBARA-II instrument, operated at FMI's Intensive Observation Area, located in Sodankylä, during the years 2009-2014.

- [1] Mike Schwank, et al. Snow Density and Ground Permittivity Retrieved from L-Band Radiometry: A Synthetic Analysis. *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* 8(8): 1-14, 2015.
- [2] Juha Lemmetyinen, et al. Snow density and ground permittivity retrieved from L-band radiometry: Application to experimental data. *Remote Sensing of Environment* 180: 377–391, 2016.

## Neural Network Kalman filtering for 3D object tracking from linear array ultrasound data

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<sup>1</sup> Research Unit of Mathematical Sciences, University of Oulu, Finland.

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Many interventional surgical procedures rely on medical imaging to visualise and track instruments. Such imaging methods not only need to be real-time capable, but also provide accurate and robust positional information. In ultrasound applications, typically only two-dimensional data from a linear array are available, and as such obtaining accurate positional estimation in three dimensions is non-trivial. In this work, we first train a neural network, using realistic synthetic training data, to estimate the out-of-plane offset of an object with the associated axial aberration in the reconstructed ultrasound image. The obtained estimate is then combined with a Kalman filtering approach that utilises positioning estimates obtained in previous time-frames to improve localisation robustness and reduce the impact of measurement noise. The accuracy of the proposed method is evaluated using simulations, and its practical applicability is demonstrated on experimental data obtained using a novel optical ultrasound imaging setup. Accurate and robust positional information is provided in real-time. Axial and lateral coordinates for out-of-plane objects are estimated with a mean error of 0.1mm for simulated data and a mean error of 0.2mm for experimental data. Three-dimensional localisation is most accurate for elevational distances larger than 1mm, with a maximum distance of 5mm considered for a 25mm aperture.

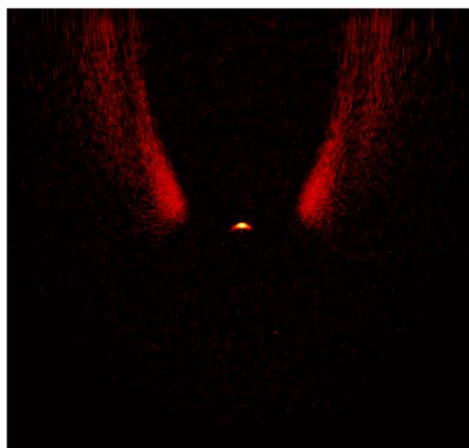


Figure 1: Optical ultrasound image reconstruction.

## Application of finite element method to General Dynamic Equation of Aerosols - comparison with classical numerical approximations

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In our research, we tested feasibility of finite element method (FEM) in approximation of general dynamic equation of aerosols (GDE), an integro-partial differential equation, which models the temporal evolution of aerosol number distribution. The GDE describes the phenomena occurring in the aerosol population: condensation/evaporation, coagulation, nucleation, and removal of the aerosol particles. [1] In addition to applying the conventional Galerkin FEM to GDE, we also introduced its extension, the Petrov-Galerkin FEM (PGFEM) to improve stability of the approximation.

The FE methods were compared to the sectional method, which is commonly used for the numerical approximation of the GDE. The sectional methods apply the 1st order difference method and size splitting operator [2] for the estimation of condensation and coagulation respectively.

The accuracy and computational effectiveness of the approximation methods were tested with cases, where analytical solution is obtained for the GDE, and with a case where time evolution approximations were compared to the time evolution of discrete GDE. The discrete GDE is an accurate representation of aerosol dynamics as the number distribution is discretized respect to the size of a monomer (molecule). [3] Analytical solutions are obtained in a case, where only condensation affects the aerosol number distribution (condensation equation) [1], in a case where only coagulation affects the number distribution (coagulation equation) [4], and in a case where both condensation and coagulation affect the number distribution [4].

The results of the numerical studies show that in the case of the condensation equation, the FEM and PGFEM were more accurate than the sectional method. The adoption of the PGFEM reduced numerical oscillation that appeared in the numerical approximation when the number of elements was low. In the case of the coagulation equation, the sectional method was more accurate than the FEM. Finally, in the cases where the evolution of the aerosol number distribution was driven by both condensation and coagulation processes, the FE methods were more accurate than the sectional method. These results indicate that the FEM and PGFEM are viable methods for estimating the temporal evolution of the aerosol number distribution.

- [1] J. H. Seinfeld, S. N. Pandis, Atmospheric Chemistry and Physics, from Air Pollution to Climate Change, John Wiley & Sons, New York, 2006.
- [2] K. E. Lehtinen, M. R. Zachariah, Self-preserving theory for the volume distribution of particles undergoing Brownian coagulation, *Journal of Colloid and Interface Science* 242 (2), 2001.
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- [4] T. W. Peterson, F. Gelbard, & J. H. Seinfeld, Dynamics of source-reinforced, coagulating, and condensing aerosols. *Journal of Colloid And Interface Science*, 63(3), 1978.

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Acknowledgements: This research was funded by the Maj and Tor Nessling foundation, the Academy of Finland (Centre of Excellence of Inverse Modelling and Imaging, 2018-2025, project 303801 and project 325647), and the Flagship Programme Atmosphere and Climate Competence Center (ACCC), 2020-2023, project 337550.

## Sensitivity of radiance simulations over snow for satellite-based remote sensing of carbon dioxide

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To meet the climate change mitigation targets, global greenhouse gas emission monitoring is needed. In the Arctic and boreal regions, continuous monitoring can inform on the expanding anthropogenic activities and their emissions as well as the changing carbon cycle and natural carbon dioxide (CO<sub>2</sub>) uptake in the changing climate. However, in the high latitudes there are numerous properties making observations difficult: large solar zenith angles, frequent cloud cover and snow.

Observing the CO<sub>2</sub> from a satellite is an ill-posed inverse problem. The solar radiation reflected from the Earth surface is measured by a satellite instrument and the amount of CO<sub>2</sub> is inferred from the attenuation. Parameters, such as other absorbing atmospheric gases, scattering from air molecules and aerosols and the reflectivity of the Earth's surface also need to be estimated.

ESA SNOWITE is a feasibility study funded by European Space Agency for examining how to improve satellite-based remote sensing of CO<sub>2</sub> over snow-covered surfaces. Snow surfaces are very dark in the near infrared bands used in CO<sub>2</sub> observations, but they have a considerable forward-reflecting peak. This supports the fact that improvements could be attained with modified observation geometry.

In this poster, the effects of varying CO<sub>2</sub> concentrations in different atmospheric layers on the simulated radiance spectra is examined in nadir and glint satellite observation modes over snow-covered surfaces.



## Spectral analysis of the stochastic Helmholtz equation in the plane

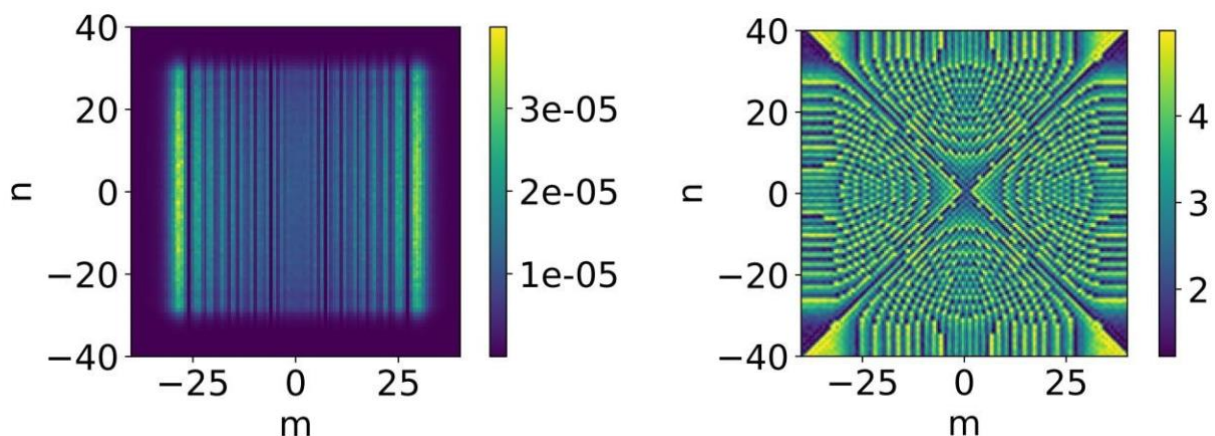
Kristoffer Linder-Steinlein<sup>1</sup> and Mirza Karamehmedović<sup>1</sup>

<sup>1</sup> Technical University of Denmark, Dept. of Applied Mathematics and Computer Science, Kgs. Lyngby, Denmark

We consider time-harmonic acoustic or transverse-electric (TE) electromagnetic waves in the plane at fixed frequency modelled by the Inhomogeneous Helmholtz equation. The model is then extended by letting various quantities behave stochastically; here, the focus is on the case where the medium is sporadically changing.

The idea was to follow the approach outlined in [1], where a singular value decomposition of the forward operator belonging to the Inhomogeneous Helmholtz equation in a vacuum, and a purely deterministic setting, was established, with a similar result proved in [2] for the three-dimensional setting. To obtain this operator, the corresponding Lippmann-Schwinger equation was derived and expanded in terms of its Neumann series under restrictions ensuring convergence.

It was possible to determine the spectrum for the normal operator to the normal equations of the system in the presence of a random medium. For this operator, it was found that the introduction of simple random fields leads to a spectral leakage. This spectral leakage did not only alter the amplitude of the singular values, but it introduced a dispersion as the angle of the spectral values were no longer purely real and positive.



**Figure 1:** Illustration of the mean magnitude and phase of the the spectral values for the normal operator linked to the stochastic Helmholtz equation.

- [1] Mirza Karamehmedović. Explicit tight bounds on the stably recoverable information for the inverse source problem. *Journal of Physics Communications*, 2: 095021 (14 pp.), 2018. ISSN 23996528.
- [2] Adrian Kirkeby, Mads T. R. Henriksen, and Mirza Karamehmedović. Stability of the inverse source problem for the Helmholtz equation in  $\mathbb{R}^3$ . *Inverse Problems*, 36, 2020.

## Numerical Modelling for the Juventas Radar (JuRa) investigation: Analysis of simulated full-wave scattering data for test fragments in time domain

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<sup>1</sup> Computing Sciences, Faculty of Information Technology and Communication Sciences, Tampere University, Korkeakoulunkatu 3, FI-33014 Tampere, Finland.

Juventas Radar (JuRa) [1] will perform a tomographic radar investigation as a part of European Space Agency's coming planetary space mission HERA [2], which will rendezvous its target asteroid 65803 Didymos in 2027. JuRa's goal is to find out the internal structure of the 160 m diameter Dimorphos, the asteroid moon of Didymos. This presentation concerns preliminary results on modelling full-wave radar signals in time domain [3] for numerical test fragments that have been designed to match the parameters of the JuRa investigation. Due to the payload limitations and other mission constraints the wavelength is relatively short compared to the diameter of the target while the measurement point configuration is might be sparse. Full-wave modelling is important to minimize any modelling errors which can significantly affect the inferences made based on sparse data. When a full-wave model is applied, the relatively short wavelength, however, sets a challenge from the viewpoints of the computational cost and accuracy of the numerical simulation. The results suggest that the 60 MHz center frequency and 20 MHz bandwidth of JuRa can be applied in a volumetric time-domain numerical simulation when a state-of-the-art computing cluster and an appropriate level of parallelization are applied. Large-memory GPUs provide a potential solution to perform the simulations effectively for a large number of time points. These preliminary results have been obtained in collaboration with the HERA workgroups preparing the mission.

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- [2] Michel, Patrick, et al. "European component of the AIDA mission to a binary asteroid: Characterization and interpretation of the impact of the DART mission." *Advances in Space Research* 62.8 (2018): 2261-2272.
- [3] Sorsa, Liisa-Ida, et al. "Bistatic full-wave radar tomography detects deep interior voids, cracks, and boulders in a rubble-pile asteroid model." *The Astrophysical Journal* 872.1 (2019): 44.

## Fully Bayesian approaches in M/EEG source imaging: an introduction

Alberto Sorrentino<sup>1</sup>, Gianvittorio Luria<sup>1</sup>, Sara Sommariva<sup>1</sup>, Alessandro Viani<sup>1</sup>

<sup>1</sup> Dipartimento di Matematica, Università degli Studi di Genova, Italy

Source imaging in magneto/electro-encephalography amounts to estimating the neural generators of the measured magnetic field/electric potential recorded at the scalp. This inverse problem is typically stated as an underdetermined, linear inverse problem whose solution is (the discretization of) a continuous current distribution inside the brain. However, in many contexts it is equally effective to assume that the neural generators are a set of point sources, whose number and locations have to be estimated. In this case, the solution is a relatively low-dimensional object for which full Bayesian inference can be done.

In this talk I will describe SESAME, a Bayesian approach introduced a few years ago [1,2] that provides a Monte Carlo approximation to the full posterior distribution of a multi-dipole model, including a posterior distribution on the number of dipoles. We present the main ideas and discuss advantages and pitfalls. I will show that the method can effectively recover complex source configurations [3]. This presentation also serves as an introduction to the subsequent talk by Alessandro Viani, who will describe the most recent developments of the method.

- [1] Sorrentino, A., Luria, G. and Aramini, R., 2014. Bayesian multi-dipole modelling of a single topography in MEG by adaptive sequential Monte Carlo samplers. *Inverse Problems*, 30(4), p.045010.
- [2] Sommariva, S. and Sorrentino, A., 2014. Sequential Monte Carlo samplers for semi-linear inverse problems and application to magnetoencephalography. *Inverse Problems*, 30(11), p.114020.
- [3] Viani, A., Luria, G., Bornfleth, H. and Sorrentino, A., 2021. Where Bayes tweaks Gauss: Conditionally Gaussian priors for stable multi-dipole estimation. *Inverse Problems and Imaging*, 15(5), p. 1099.

## Selection and averaging of hyper-parameter comes for free with SMC samplers

Alessandro Viani<sup>1</sup>, Alberto Sorrentino<sup>1</sup>

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We present a powerful method for performing hyper-parameter selection and averaging for free in Bayesian inverse problems through the usage of Sequential Monte Carlo (SMC) Samplers.

SMC samplers represent largely used methods in Bayesian inference for the approximation of complex probability distributions, based on the sequential approximation of a sequence of more and more complex densities, the first one being easy to sample, and the last one being the target density. We construct an SMC sampler for a class of Bayesian inverse problems where the likelihood is in a Natural Exponential Family (NEF) with a scalar hyper-parameter in such a way that every density of the - tempering - sequence can be interpreted as a posterior density properly conditioned on a different value of the hyper-parameter. This approach allows, without any additional computational cost, a wise selection of a specific value for the hyper-parameter or its marginalization, therefore recycling SMC samplers particles.

The most obvious application of the proposed method is the approximation of the joint posterior distribution in hierarchical Bayesian inverse problems for both the unknowns and the hyper-parameter such as in source analysis of Magneto/Electro-Encephalography (MEG) data, where the noise standard deviation plays the role of the hyper-parameter.

For analyzing the performances of the algorithm we provide an example of a conditionally linear Gaussian problem where analytical solutions are available; showing that the proposed approach is essentially better than the traditional approach where the hyper-parameter is sampled by the SMC sampler.

Concluding, we show that the proposed approach provides reliable results and a substantial reduction of computational cost with respect to alternative approaches.

## Lasso-based estimation of the cross-power spectrum in linear inverse problems and its application to functional brain connectivity.

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Let's consider the problem of obtaining sparse estimates of the cross-power spectrum of a multivariate stochastic process,  $\mathbf{x}(t) = (x_1(t), \dots, x_n(t))^T$ , that can be only observed indirectly through another stochastic process,  $\mathbf{y}(t) = (y_1(t), \dots, y_m(t))^T$ . Specifically, let's assume  $\mathbf{x}(t)$  and  $\mathbf{y}(t)$  to be related by a linear inverse problem with additive Gaussian noise, i.e.  $\mathbf{y}(t) = \mathbf{G} \mathbf{x}(t) + \mathbf{e}(t)$ , where  $\mathbf{G}$  is the gain matrix of size  $m \times n$ , and  $\mathbf{e}(t) = (e_1(t), \dots, e_m(t))^T$  represents measurement noise affecting the observations.

In the first part of this talk, I will show how, by exploiting the linearity of the Fourier Transform, it is possible to derive a linear inverse problem relating the cross-power spectrum of the unknown,  $\mathbf{S}_x(f)$ , to that of the data,  $\mathbf{S}_y(f)$  [1]. Then I will show how to obtain sparse estimates of  $\mathbf{S}_x(f)$  by using Lasso regularization and the FISTA (Fast Iterative Shrinkage-Thresholding Algorithm) algorithm [2].

The reliability and relevance of the proposed method will be demonstrated through its application for estimating functional brain connectivity from magnetoencephalographic data.

[1] Vallarino Elisabetta, Sommariva Sara, Piana Michele, Sorrentino Alberto. On the two-step estimation of the cross-power spectrum for dynamic linear inverse problems. *Inverse Problems* 36(4):045010, 2020

[2] Beck Amir and Teboulle Marc. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM journal on imaging sciences*, 2(1):183-202, 2009.

## Helsinki Deblur Challenge 2021: Deep Learning Approach with Conditional Blur Level Information

Ji Li<sup>1</sup>, Weixi Wang<sup>1</sup>, Ziyi Yang<sup>1</sup>

<sup>1</sup> Department of Mathematics, National University of Singapore, 21 Lower Kent Ridge Rd, 119077, Singapore.

We presented our deep learning approach for the Helsinki Deblur Challenge 2021 organized by Finnish Inverse Problems Society (FIPS). In our approach, we borrowed the DeblurGAN-v2 network architecture with special modification for the competition, including the conditional blur information integrated into the network, the new coarse-to-fine training technique. All the modifications will be discussed. Different from other solutions, we aim to use a universal network to deblur the input with different blur level. In this way, the training time is reduced. Our solution goes to 2<sup>nd</sup> place among all the solutions.

## Using mixed-scale dense CNNs, multiple scales, and simulated training data for the Helsinki Deblur Challenge 2021

Daniël M. Pelt<sup>1</sup>

<sup>1</sup> LIACS, Leiden University, Niels Bohrweg 1, 2333 CA Leiden, The Netherlands

This talk is about a submission to the Helsinki Deblur Challenge 2021 [1] which came in 3<sup>rd</sup> place. The approach uses Mixed-Scale Dense CNNs [2] to deconvolve images of text. A separate set of networks is trained for each blur category, but for all categories the used networks and training approaches are identical. A multi-scale approach is used, in which five networks are trained for each blur category: first, a network is trained at 20x downscaled images, then, a network is trained at 10x downscaled images, and at 4x, 2x, and no downscaling afterwards. At each scale, an upscaled version of the network output of the previous scale is used as additional input.

The key to improving results for large blur categories was to create a large set of simulated training images using the provided images for other blur levels. So for each blur category, we first trained a multi-scale set of five networks to simulate the blurring by using the provided unblurred images as input and blurred images as training target. The trained networks were then applied to the 3800 unblurred images of all other blur levels to produce simulated virtual blurred images. A new multi-scale set of five networks was then trained using the virtual blurred images as input and the unblurred images as target to produce a deconvolving method.

In the talk, the approach will be described in detail, and results are analyzed and compared with other approaches.

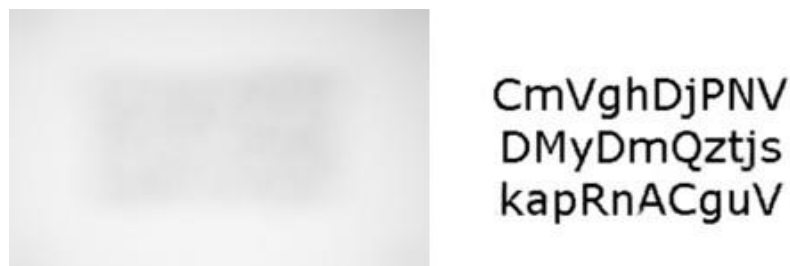


Figure 1: Input image (left) and network output (right) at blur level 17

[1] <http://fips.fi/HDC2021.php>

[2] Pelt, D. M., & Sethian, J. A. (2018). A mixed-scale dense convolutional neural network for image analysis. *Proceedings of the National Academy of Sciences*, 115(2), 254-259.

# Learning to Invert Defocus Blur: A Data-Driven Approach to the "Helsinki Deblur Challenge"

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2. Mathematical Institute, Utrecht University, Utrecht, Netherlands.

The goal of the "Helsinki Deblur Challenge" (HDC2021) was to develop a general purpose deblurring procedure to tackle the following task: Deblurring images of characters with 20 steps of gradually increasing blur. The processed images are then fed to an optical character recognition engine. The deviation of the predicted characters to the ground truth characters is then quantified using the Levenshtein distance. We call this crucial metric the "OCR score".

The network architecture building the backbone of our data driven pipeline is an adjusted version of the prominent U-Net [1]. Adding group normalisation [2], an introductory pooling layer and more down- and up-sampling steps to the CNN - the modifications allow a better performance, as the field-of-view of the network increases.

The key to our deblurring pipeline consists in the solid understanding of the blurring operator. Modelling the forward operation as a convolution followed by a moustache distortion brings several advantages: By augmenting the limited training dataset of 200 samples to an arbitrary amount of synthesized data, the networks generalization capacity is drastically improved. Furthermore, the inclusion of natural images in the training set enforces the network to become a general purpose deblurer, rather than a character classifier and generator.

Using the learned parameters of the forward model, we correct the radial distortion as a pre-processing step. This provides a substantial benefit for the subsequent U-Net processing, as it renders the problem approximately translation invariant.

Eventually, the networks are pre-trained with about 75000 samples of synthesized data and then finetuned with mainly original HDC data.

The 20 networks successfully deblur all 20 levels, reaching average OCR scores of 84.3-98.6 on our validation split and winning the HDC2021 challenge.

Blurry input	Deblurred	
QbARvsyYqD LmGrHvvJSX W WBTXNvW	QbARvsyYqD LmGrHvvJSX W WBTXNvW	original characters: QbARvsyYqD LmGrHvvJSX W WBTXNvW <b>Level 0</b>
iAJHzEkuuG DgXCuTfpus hxhtEdUnYp	iAJHzEkuuG DgXCuTfpus hxhtEdUnYp	original characters: iAJHzEkuuG DgXCuTfpus hxhtEdUnYp <b>Level 7</b>
FQeMYqKBZj QfsgKuinTr WzQRcALRLW	FQeMYqKBZj QfsgKuinTr WzQRcALRLW	original characters: FQeMYqKBZj QfsgKuinTr WzQRcALRLW <b>Level 14</b>
BBue FisKT uGNzhTHxHX spTYMuGJzrh	BBue FisKT uGNzhTHxHX spTYMuGJzrh	original characters: BBue FisKT uGNzhTHxHX spTYMuGJzrh <b>Level 19 (last)</b>

Figure 1: Example of deblurring performance.

- Ronneberger, Olaf, Philipp Fischer, and Thomas Brox. "U-net: Convolutional networks for biomedical image segmentation." International Conference on Medical image computing and computer-assisted intervention. Springer, Cham, 2015
- Wu, Yuxin, and Kaiming He. "Group normalization." Proceedings of the European conference on computer vision (ECCV). 2018



# Stability estimates in a partial data inverse boundary value problem for biharmonic operators at high frequencies

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We study the inverse boundary value problems of determining a potential in the Helmholtz type equation for the perturbed biharmonic operator

$$(\Delta^2 - k^4 + q)u = 0 \quad \text{in } \Omega \quad (1)$$

from the knowledge of the partial Cauchy data set. Here  $\Omega \subset \{x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_n < 0\}$ ,  $n \geq 3$ , be a bounded open set with  $C^\infty$  boundary. Assume that  $\Gamma_0 := \partial\Omega \cap \{x_n = 0\}$  is non-empty, and let us set  $\Gamma = \partial\Omega \setminus \Gamma_0$ . That is, our geometric setting is that of a domain whose inaccessible portion of the boundary is contained in a hyperplane, and we are given the Cauchy data set on the complement. The uniqueness and logarithmic stability for this problem were established in [2] and [1], respectively. We establish Hölder type stability estimates in the high frequency regime, with an explicit dependence on the frequency parameter, under mild regularity assumptions on the potentials, sharpening those of [1]. In particular, no continuity of the potentials in the main result is assumed, and the required Sobolev regularity assumptions are fairly mild and are independent of the dimension.

- [1] Choudhury, A., Heck, H., *Stability of the inverse boundary value problem for the biharmonic operator: logarithmic estimates*. J. Inverse Ill-Posed Probl. **25** (2017), no. 2, 251–263.
- [2] Yang, Y., *Determining the first order perturbation of a bi-harmonic operator on bounded and unbounded domains from partial data*, J. Differential Equations **257** (2014), no. 10, 3607–3639.

## Inverse scattering for quasi-linear biharmonic operator in 2D

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We consider direct and inverse scattering problems for biharmonic operator in 2D. The bi-Laplacian is perturbed by zero- and first-order quasi-linear perturbations. We discuss the solvability of the direct scattering problem and formulate the inverse problem of recovering certain combination of the perturbations from the knowledge of the scattering amplitude as scattering data. Saito's formula provides the uniqueness for this inverse problem, and in the case of full scattering data it can be directly inverted to obtain some numerical reconstructions. Finally, we will discuss back-scattering problem and present some reconstructions by using the method of Born approximation.

- [1] Harju M, Kultima J, Serov V and Tyni T, *Two-dimensional inverse scattering for quasi-linear biharmonic operator*, *Inverse Problems and Imaging*, 15(5), 1015-1033, 2021.
- [2] Kultima J and Serov V, *Reconstruction of singularities in two-dimensional quasi-linear biharmonic operator*, submitted to *Inverse Problems and Imaging*, 2021.

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Acknowledgements: This work was supported by the Academy of Finland (grant number 312123, the Centre of Excellence of Inverse Modelling and Imaging, 2018-2025). T. T. was supported by the Academy of Finland (grant number 312119, Centre of Excellence in Inverse Modelling and Imaging, 2018-2025)

## Unique determination of Anisotropic perturbations of a polyharmonic operator from partial boundary data

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We prove an inverse problem involving the unique recovery of several lower order tensor perturbations of a polyharmonic operator on a bounded domain from the knowledge of the Dirichlet to Neumann map on a part of boundary. The uniqueness proof relies on the inversion of generalized momentum ray transforms (MRT) for symmetric tensor fields. We construct suitable complex geometric optics (CGO) solutions for the polyharmonic operators that reduces the inverse problem to uniqueness results for generalized MRT.

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Acknowledgements: This work was initiated during a visit of S.B. to TIFR Centre for Applicable Mathematics, Bangalore, India, which was partly supported by a SERB MATRICS research grant of V.P.K. S.B. is partly supported by a SEED grant of Indian Institute of Science Education and Research, Bhopal, India. S.K.S. was partly supported by the European Research Council under Horizon 2020 (ERC CoG 770924).

## Uniqueness for the fractional Calderon problem with quasilocal perturbations

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We study the fractional Schrödinger equation with quasilocal perturbations. These are a family of nonlocal perturbations vanishing at infinity, which include e.g. convolutions against Schwartz functions. We show that the qualitative unique continuation and Runge approximation properties hold in the assumption of sufficient decay. Quantitative versions of both results are also obtained via a propagation of smallness analysis for the Caffarelli-Silvestre extension. The results are then used to show uniqueness in the inverse problem of retrieving a quasilocal perturbation from DN data under suitable geometric assumptions. Our work generalizes recent results regarding the locally perturbed fractional Calderon problem.

[1] G. Covi (2021). *Uniqueness for the fractional Calderón problem with quasilocal perturbations*. arXiv preprint: 2110.11063

## Optimality of increasing stability for an inverse boundary value problem

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This talk is based on my recent work [KUW21].

In [KUW21], we study the optimality of increasing stability of the inverse boundary value problem (IBVP) for the Schrödinger equation. The rigorous justification of increasing stability for the IBVP for the Schrödinger equation were established by Isakov [Isa11] and by Isakov, Nagayasu, Uhlmann, Wang of the paper [INUW14]. In [Isa11], [INUW14], the authors showed that the stability of this IBVP increases as the frequency increases in the sense that the stability estimate changes from a logarithmic type to a Hölder type. In [KUW21], we prove that the instability changes from an exponential type to a Hölder type when the frequency increases. This result verifies that results in [Isa11], [INUW14] are optimal.

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[KUW21] P.-Z. Kow, G. Uhlmann, and J.-N. Wang. Optimality of increasing stability for an inverse boundary value problem. arXiv preprint, 2021. [arXiv:2102.11532](#), to appear in *SIAM Journal on Mathematical Analysis (SIMA)*.

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Acknowledgments: Kow and Wang were partly supported by MOST 108-2115-M-002-002-MY3 and 109-2115-M-002-001-MY3. Uhlmann was partly supported by NSF, a Walker Family Endowed Professorship at UW and a Si-Yuan Professorship at IAS, HKUST.

## An inverse problem for a semi-linear wave equation

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<sup>3</sup> Department of Mathematics, University of Toronto, Toronto, Canada.

In this talk we consider the recovery of an unknown potential associated with a semi-linear wave equation from Dirichlet lateral boundary measurements on a bounded time-cylinder. The framework is the  $n+1$  dimensional Euclidean space. We shall show that the potential can be stably recovered with a modulus of continuity of Hölder type. Our proof is constructive, and it is based on the higher-order linearization method [1]. Consequently, we also get uniqueness results; that is, the lateral boundary measurements uniquely determine the unknown potential. The talk will be based on the results of the work [2].

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Acknowledgements: M.L was supported by the Academy of Finland, grants 320113, 318990, and 312119. L. P-M and T.L. were supported by the Academy of Finland (Centre of Excellence in Inverse Modeling and Imaging, grant numbers 284715 and 309963) and by the European Research Council under Horizon 2020(ERC CoG 770924). T. T. was supported by the Academy of Finland (Centre of Excellence in Inverse Modeling and Imaging, grant number 312119).

## Reconstructing subcortical and cortical activity of Somatosensory via Inverse modeling techniques utilizing SEP datasets

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In this study, we investigate reconstructing the sequential components of somatosensory evoked potentials (SEPs) at different latencies between 14-30 ms. Detecting subcortical activity is a challenging task since subcortical structures are far from the sensors on the scalp, however, it plays a significant role in treatment of brain disorders such as Alzheimer's disease or refractory epilepsy. To this end, we applied randomized multiresolution scanning (RAMUS) [1] technique in the framework of hierarchical Bayesian, conditionally Gaussian approach which is recently proposed as a potential technique to reconstruct cortical and simultaneous subcortical activity. Our goal is to detect the simultaneous cortical and weak deep activity in reconstructing the generator of SEP components of median nerve stimulation for earlier and late components. For our analysis, we used three different realistic head models and experimental measurement datasets. RAMUS is a maximum a posteriori estimation technique aiming to reduce optimization and discretization errors via randomized source spaces and coarse to fine reconstruction strategy. The source space is decomposed into random subsets during a refinement process which is repeated for several randomized configurations and the final reconstruction is obtained as an average over the multiple resolution levels. RAMUS found weakly distinguishable deep activity for earlier components while simultaneous cortical and subcortical activity was detected for components after 20 ms. The earliest SEP component, P14/N14, was reconstructed at medial lemniscus [2] and SEP component, P20/N20, was localized at Brodmann area 3b [3]. In order to compare the performance of RAMUS, we applied other inversion techniques such as sLORETA and Beamformer for reconstructing the SEP components at aforementioned latencies. Our findings reveal RAMUS as a promising method for localizing cortical and weakly distinguishable deep activity in the case of realistic and multicompartiment head models and experimental measurements.

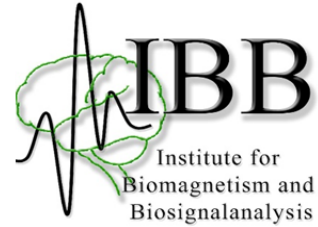
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## New non-invasive multimodal neuroimaging and neurostimulation methods for improved diagnosis and therapy in refractory focal epilepsy

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In recent years, the use of electroencephalography (EEG) and magnetoencephalography (MEG) source imaging (ESI, MSI) has gained considerable attention in presurgical epilepsy diagnosis. The source imaging is generally combined with further modalities such as video-EEG, MRI, neuropsychology and others.

In my talk, I will discuss new techniques of forward and inverse modeling in ESI and MSI and especially combined E/MSI of ictal and interictal epileptic activity. I will present data on how the source analysis results are discussed and interpreted in the context of the accompanying further modalities. Case studies will be presented that enlighten the use of this multimodal diagnostic procedure in epileptology and evidence on clinical value is summarized. Finally, an accurate diagnosis enables also new therapy approaches where I will focus on non-invasive targeted and optimized multi-channel transcranial electric stimulation.





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## A Cut finite element method for the EEG forward problem

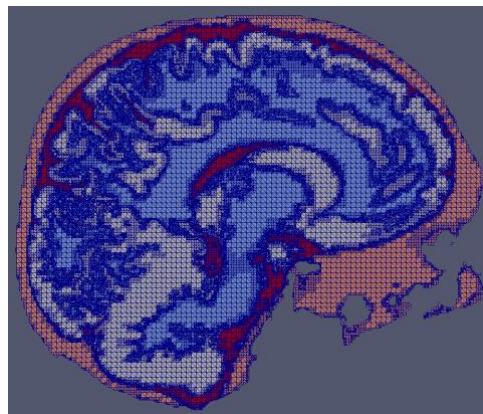
Tim Erdbrügger<sup>1</sup>

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When using the finite element method to model the head as a volume conductor, one usually has the choice between tetrahedral and hexahedral mesh generation. Both have distinct advantages and disadvantages when it comes to their accuracy or ease of implementation. Hexahedral meshes for example cannot properly follow the curvature of the folded brain structures while tetrahedral mesh generators impose strong conditions on the surface triangulations they are created from, rendering them difficult to create automatically .

CutFEM and other unfitted FEM approaches are well suited to alleviate these and other concerns. Rather than creating a mesh that is specifically tailored to the head, it uses a geometry-independent background mesh. The mesh is then cut into pieces by level set functions that outline the surfaces of the different head compartments.

After briefly introducing the method, comparisons in sphere models, where analytical solutions exist, will be shown as well as early results for realistic cases.



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## Validating EEG, MEG and combined MEG and EEG beamforming for an estimation of the epileptogenic zone in two focal cortical dysplasia epilepsy patients

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MEG and EEG source analysis is frequently used for the presurgical evaluation of pharmacoresistant epilepsy patients. The source localization of the epileptogenic zone depends, among other aspects, on the selected inverse and forward approaches and their respective parameter choices.

We compare the standard dipole scanning method with two beamformer approaches and we investigate the influence of the covariance estimation method and the strength of regularization on the localization performance for EEG, MEG and combined EEG and MEG.

We compare the difference between six-compartment and standard three-compartment head modeling. In a retrospective study of two patients with focal epilepsy due to focal cortical dysplasia type IIb we used the distance of the localization of interictal epileptic spikes to the surgery resection as reference for good localization. We found that beamformer localization can be sensitive to the choice of the regularization parameter, which has to be individually optimized. Estimation of the covariance matrix with averaged spike data yielded more robust results across the modalities. For appropriate regularization parameter choices, the beamformer localized better than the standard dipole scan. Compared to the importance of an appropriate regularization, the sensitivity of the localization to the head modeling was smaller.

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Acknowledgements: This research was supported by the German Research Foundation (DFG) through projects WO1425/7-1 and RA2062/1-1 and by the Bundesministerium für Gesundheit (BMG) as project ZMI1-2521FSB006, under the frame of ERA PerMed as project ERAPERMED2020-227 PerEpi.



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## Efficient Computation of Transfer Matrices using the Block Conjugate Gradient Method

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To estimate neural activity from measurements of electromagnetic fields at the head surface it is necessary to compute the potentials and magnetic fluxes generated by many different source configurations. For example, inversion approaches using a Hierarchical Bayesian Model (HBM) often consider more than 10000 different source configurations. This can be done efficiently using the *Transfer Matrix Approach*. Once the so called *Transfer Matrix* is computed, calculating potentials and fluxes reduces to a simple matrix-vector multiplication, instead of e.g. using a Finite Element process. While using the transfer matrix is efficient, computing the Transfer Matrix takes a significant amount of time. It is thus of interest to make this computation as efficient as possible.

To compute the Transfer Matrix, one needs to solve a linear system

$$Ax_i = b_i$$

for every sensor position, where we have the same linear operator  $A$  for every system. These systems are often solved by using Krylov methods on each system individually. Another approach is to view the linear systems as a single matrix equation

$$AX = B,$$

and solve this equation using a so called Block Krylov method. In this talk, we want to give an introduction to a new Block Krylov framework. We further show that Block Krylov methods can lead to a substantial speed up for spherical head models as well as for realistic head models.

## Electrical capacitance tomography assisted control in a microwave drying process

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Microwave drying is a drying technology with volumetric and selective heating advantages and has different applications, such as drying polymer foams after the impregnation process in the thermal insulation industry [1]. One of the main goals in the microwave drying process of polymer foams is to reach a certain amount of moisture (e.g., 10%) after the drying process. Designing a moisture controller for this process can help to achieve this goal. However, the moisture controller requires moisture data from the polymer foams, and since the foam is constantly moving on a conveyor belt, the use of conventional methods is impossible.

An electrical capacitance tomography (ECT) sensor is an attractive tool to estimate and monitor moisture as it is contactless and fast [2]. In this research, an ECT sensor is designed and installed at the exit of the microwave oven to reconstruct the permittivity distribution of polymer foam after passing through the oven, as shown in Fig. 1. The effective permittivity of the foam is correlated to its moisture content, so by controlling the permittivity, the moisture is controlled. The moisture information is sent to the controller, and the controller adjusts the power level of the microwave sources based on the desired permittivity/moisture (set point), such that the output foam moisture follows the set point.

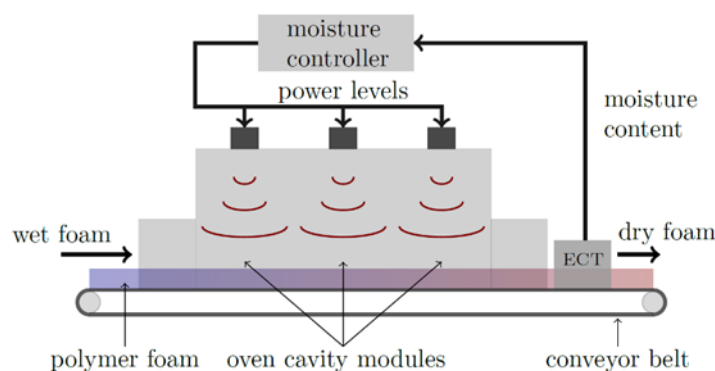


Figure 1: A schematic of the ECT-assisted control of microwave drying process.

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Acknowledgements: This research was supported in part by the European Union's Horizon 2020 Research and Innovation Program through the Marie Skłodowska-Curie Grant under Agreement 764902 and in part by the Academy of Finland under Project 312344 (Finnish Centre of Excellence of Inverse Modeling and Imaging) and Project 321761.

## Hierarchical Deconvolution for Incoherent Scatter Radar Data

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We propose a novel method for deconvolving incoherent scatter radar data to recover accurate reconstructions of backscattered powers. The problem is modelled as a hierarchical noise-

perturbed deconvolution problem, where the lower hierarchy consists of an adaptive length-scale function that allows for a non-stationary prior and as such enables adaptive recovery of smooth and narrow layers in the profiles. The estimation is done in a Bayesian statistical inversion framework as a two-step procedure, where hyperparameters are first estimated by optimisation and followed by an analytical closed-form solution of the deconvolved signal.

The proposed optimisation-based method is compared to a fully probabilistic approach using Markov Chain Monte Carlo techniques enabling additional uncertainty quantification. We examine the potential of the hierarchical deconvolution approach using two different prior models for the length-scale function.

We apply the developed methodology to compute the backscattered powers of measured Polar Mesospheric Winter Echoes, as well as Summer Echoes, from the EISCAT VHF radar in Tromsø, Norway. Computational accuracy and performance are tested using a simulated signal corresponding to a typical background ionosphere and a sporadic E layer with known ground-truth. The results suggest that the proposed *hierarchical deconvolution* approach can recover accurate and clean reconstructions of profiles, and the potential to be successfully applied to similar problems.

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Acknowledgements: This work was supported by Academy of Finland Profi 5 (HiDyn) funding for mathematics and AI: data insight for high-dimensional dynamics (grant 326291), and by Profi 2 funding (grant 301542), and Academy of Finland projects 336787, 336796, 338408. We acknowledge the EISCAT Association for the incoherent scatter radar data used in this study.

## Testing the topological derivative against experimental data; the Institut Fresnel database

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Inversion methods based on the computation of the topological derivative [5] of a mismatch functional have been devised for a wide range of phenomena. These methods have been tested against numerical data with very good results [3,4]. However, there are very few papers reporting on its performance with experimental data.

The Institut Fresnel kindly published several databases [1] intended for the testing of inversion methods against microwave experimental data. The purpose of these databases is twofold. On the one hand it gives research groups without experimental facilities a way of testing their methods and on the other hand it serves as some sort of benchmark for different inversion methods.

In [2] we applied inversion methods based on the computation of the topological derivative or energy to the Fresnel databases with highly satisfactory reconstructions, as seen in figure 1. In this talk we will present those results as well as several possible improvements which are under study.

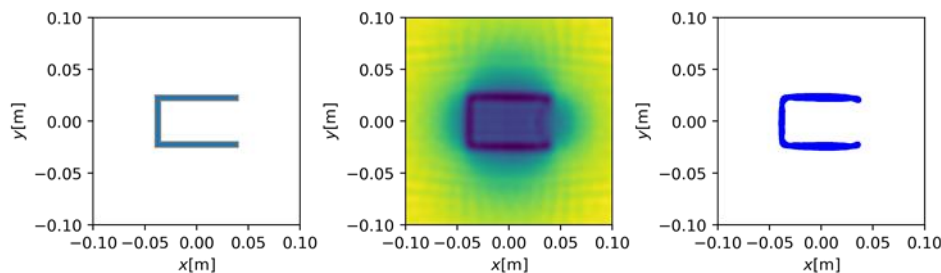


Figure 1: object (left), topological derivative (center) and reconstruction (right).

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- [2] Ana Carpio, Manuel Pena and María-Luisa Rapún. Processing the 2D and 3D Fresnel experimental databases via topological derivative methods. *Inverse Problems*, 37(10):105012, 2021.
- [3] Ana Carpio and María-Luisa Rapún. Topological derivatives for shape reconstruction. *Inverse Problems and Imaging*, 1943: 85-133, 2008.
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- [5] Jan Sokolowski and Antoni Zochowski. On the topological derivative in shape optimization. *SIAM Journal on Control and Optimization*, 37(4): 1251-1272, 1999.

Acknowledgements: This work was supported by Spanish FEDER/MICINN-AEI grants TRA2016-75075-R, MTM2014-56948-C2-1-P and grant PID2020-114173RB-I00 funded by MCIN/AEI/ 10.13039/501100011033

## Breaking cosmological conformal gauge with neutrinos

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It is an old inverse problem to look up in the sky and want to construct an accurate description of the universe. In the context of general relativity, this problem is, perhaps surprisingly, solvable with purely geometric data without any spectral information.

Most cosmological measurements are done with photons, but that leaves an unfortunate gauge symmetry: The data is invariant under conformal transformations of the model, but the conformal factor is physically meaningful. This conformal gauge can be broken with neutrinos, and geometric cosmology using both photons and neutrinos leads to full uniqueness.

The talk is based on joint work with Gunther Uhlmann.

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Acknowledgements: This work was supported by Academy of Finland.

# An inverse problem for the minimal surface equation

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In this work we study a novel inverse problem for the minimal surface equation, which is a quasilinear elliptic PDE. Consider a Riemannian manifold  $(M, g)$  where  $M = \mathbb{R}^n$  and the metric is conformally Euclidean i.e.  $g_{ij}(x) = c(x)\delta_{ij}$ . Let  $u: \Omega \subset \mathbb{R}^{n-1} \rightarrow \mathbb{R}$  be a smooth function that satisfies the minimal surface equation. Assume that we can make boundary measurements on the graph of  $u$ , that is we know the Dirichlet-to-Neumann (DN) map which maps the boundary value  $u|_{\partial\Omega} = f$  to the normal derivative  $\partial_\nu u|_{\partial\Omega}$ . The Dirichlet data  $f$  is the height of minimal surface on the boundary. The normal derivative  $\partial_\nu u|_{\partial\Omega}$  can be thought of as tension on the boundary caused by the minimal surface. In this talk we show that from the knowledge of the DN map we can determine information about the conformal factor. In particular, we can determine  $\partial_{x_n}^k c(x', 0)$  for  $k = 0, 1, \dots$ ,  $x' \in \mathbb{R}^{n-1}$ . We use the technique of higher order linearization (see for example [1]) that has received increasing attention lately.

[1] Lassas, M., Liimatainen, T., Lin, Y.-H., Salo, M, Inverse problems for elliptic equations with power type nonlinearities. *J. Math. Pures Appl.*, (9) 145 (2021), 44–82.



## Pestov identities and X-ray tomography on manifolds with non-smooth Riemannian metrics

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How regular does a Riemannian metric have to be for the geodesic X-ray transform to be injective? In this talk, we consider the Pestov energy identity method for proving injectivity results for the geodesic X-ray transform. We show that the same identity holds true on manifolds equipped with  $C^{1,1}$  regular Riemannian metrics and we prove injectivity results on Riemannian manifolds we call simple  $C^{1,1}$  manifolds. When the Riemannian metric is  $C^\infty$ -smooth the definition of a simple  $C^{1,1}$  manifold is equivalent to the traditional definition of a simple manifold. This is joint work with Joonas Ilmavirta.

# Sparse Dynamic Tomography Regularization with 4D Representation Systems

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Due to developments on many fronts, computed tomography (CT) has become accessible and viable method for gaining information about the interior structure of a wide variety of objects. This has lead to interest in applications where the measurement setup or the unknown target are less than optimal for traditional reconstruction methods. For example studying nutrient perfusion in plants with X-ray CT leads to undersampled measurements from a changing object [1].

To overcome the challenges of this ill-posed inverse problem, specialized, motion aware and computationally reasonable regularization methods are necessary. Especially if the evolution in time needs to be studied in all 3 spatial dimensions.

Sparsity based regularization methods with wavelets and shearlets are well known to the inverse problems community and have provided many encouraging results in the past. In addition they are widely used in other applied mathematics and signal processing problems when dealing with multivariate data with specific geometric features. In most applications, the key idea is that the representation system should efficiently capture the underlying characteristics and singularities – which in higher dimensions can take various forms.

Out of the many possibilities I wish to highlight two: the 4D dual-tree complex wavelets [2] which provide the computational efficiency of real-valued wavelets with few improvements especially in higher dimensions, and 4D cylindrical shearlets [3] which are designed for data dominated by edge-like singularities in 3 (spatial) dimensions only.

[1] T. A. Bubba, T. Heikkilä, H. Help, S. Huotari, Y. Salmon and S. Siltanen, "Sparse dynamic tomography: A shearlet-based approach for iodine perfusion in plant stems" (2020). *Inverse problems*, 36(9), 094002.

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## High sensibility imaging of defects in waveguides using near resonance frequencies

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Localization and reconstruction of defects in waveguides is of crucial interest in nondestructive evaluation of structures. This work aims to present a new multi-frequency inversion method to image shape variations in slowly varying waveguides, based on physical experiments done at the Institut Langevin [1]. Both mechanic or electromagnetic guides can be considered in 2 or 3 dimensions. Contrary to previous works on this subject [2], we choose to take advantage of the near resonance frequencies of the waveguide, where the Helmholtz problem is known to be ill-conditioned.

At these frequencies, a phenomenon close to the tunnel effect in quantum mechanics can be observed (see Figure 1), and resonant modes propagate in the waveguide [3]. These resonant modes are very sensitive to width variations, and measuring their amplitude enables us to reconstruct the local variations of the waveguide shape with very high sensibility. Given wave field surface measurements for a range of near resonance frequencies, we provide an L2 stable reconstruction of the width of a slowly varying waveguide.

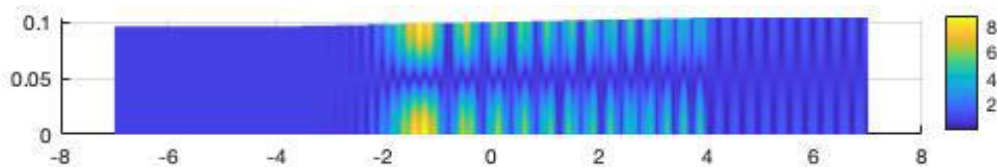


Figure 1: Representation of the amplitude of the wavefield near resonance frequency

We also provide a suitable numerical method for efficient reconstruction of such width variations, and some numerical reconstructions can be found in Figure 2. We finally compare our results to other methods, such as [2].



Figure 2: Reconstruction of two slowly varying waveguides. In black, the initial shape, in red, the reconstruction slightly shifted for comparison purposes.

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- [3] R. B. Nielsen and N. Peake. Tunneling effects for acoustic waves in slowly varying axisymmetric flow ducts. *Journal of Sound and Vibration*, 380:180-191, 2016.

## Experimental and Computational Resources for Computed Tomography Research at the University of Helsinki

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Computed tomography is an imaging modality in which X-ray projections taken from different directions around an object are combined to form cross-sectional images by using a reconstruction algorithm [1]. Significant theoretical and applied research in these algorithms has been conducted in the FIPS research network. However, there still remains a significant gap between this basic research and putting the algorithms into practical use with real data and applications.

This presentation will give an overview of the experimental and computational resources developed at the University of Helsinki to bridge this gap. Experimental resources include a custom-built cone beam computed tomography scanner specifically designed for collecting datasets for the development of CT algorithms, with the ability to conduct measurements using either a conventional energy-integrating X-ray detector or a photon counting detector. Computational tools include HelTomo, a Matlab toolbox specifically designed to be used with X-ray datasets gathered in X-ray laboratories, providing an easy yet flexible user interface, designed for researchers to be able to focus on the essential: computing both 2D and 3D CT reconstructions and developing new reconstruction algorithms.

[1] Geoffrey D. Rubin. Computed tomography: revolutionizing the practice of medicine for 40 years. *Radiology*, 273(2 Suppl), 2014.

## Randomized Sampling Methods for Linear Inverse Problems

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In recent years, randomized methods have received considerable attention since they can be used to reduce the dimensionality of large linear systems. Since the late 1980s, they are a well-established tool to efficiently solve high-dimensional computational problems in computer science. At the end of the 1990s, random methods were firstly applied to find low-rank approximations of matrices with a low computational cost (see [2] for a review). More recently, they were used also for the solution of linear inverse problems ([3], [4]).

In this talk I will give a brief overview on the use of randomized sampling methods for the solution of linear inverse problems. Then, I will present some recent results on the solution of the magnetoencephalography/electroencephalography (MEG/EEG) inverse problem by the random sampling method [1]. The sampling procedure reduces the dimensionality of the source space so that the dimensionality of the MEG/EEG inverse problem is reduced accordingly. Numerical simulations on synthetic and real data show that the reconstruction error of the method is comparable to that of classical inversion methods with the advantage to have a reduced computational load.

The sampling procedure is proved to be a very simple technique that can be easily implemented on portable devices. For this reason, randomized sampling methods are very attractive when addressing real-time applications, like noninvasive brain-machine interface or neuro-feedback rehabilitation, where fast and slim inversion techniques are recommended.

This is a joint work with L. Della Cioppa, A. Pascarella, M. Tartaglione.

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# Fast 3D CGO-Based Reconstruction for Absolute Electrical Impedance Tomography on Experimental Data from ACT5

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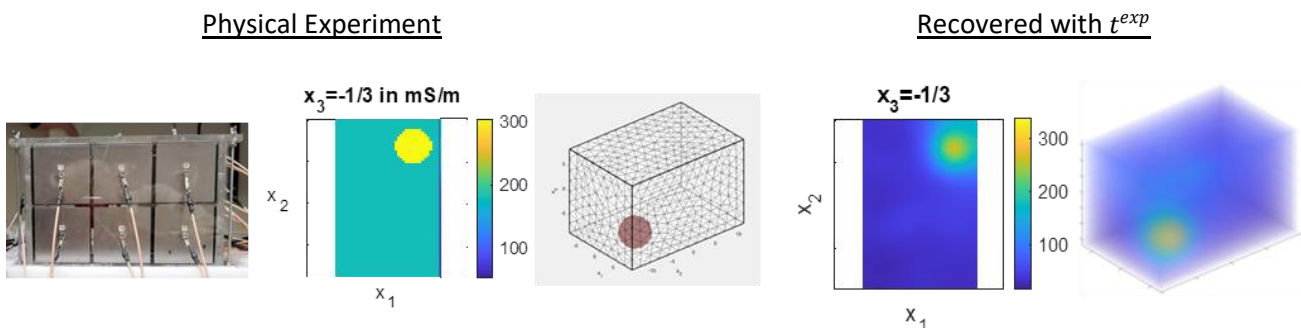
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Absolute EIT image reconstruction is a severely ill-posed nonlinear inverse problem that requires carefully designed algorithms that are robust to noise and modelling errors. Optimization-based methods provide an avenue but are computationally demanding, especially in 3D where the forward problem is solved repeatedly on a fine FEM grid. Alternatively, CGO-based methods have been shown to be quite robust and real-time capable in 2D [1]. Recently, the  $t^{exp}$  and Calderón CGO-based methods have been implemented for full 3D reconstruction using simulated electrode data [2] building upon the work of [3].

In this talk, we demonstrate the methods on experimental EIT data collected on the ACT5 system [4] using 32 electrodes attached to a plexiglass box. Agar targets with known conductivity values were used. The CGO-methods, while not fully optimized for speed, took an average of 5 seconds for the  $t^{exp}$  approach, and under a second for Calderón, potentially putting real-time 3D absolute EIT within reach. By comparison, iterative methods often take on the order of tens of minutes or hours.



- [1] Melody Dodd and Jennifer L. Mueller. A Real-time D-bar Algorithm for 2-D Electrical Impedance Tomography Data. *Inverse Problems and Imaging*, 8(4):1013–1031, 2014.
- [2] Sarah J. Hamilton, David Isaacson, Ville Kolehmainen, Peter A. Muller, Jussi Toivanen, and Patrick F. Bray. 3D Electrical Impedance Tomography reconstructions from simulated data using direct inversion  $t^{exp}$  and Calderón methods. *Inverse Problems and Imaging*, 15(5):1135–1169, 2021.
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- [4] Omid Rajabi Shishvan, Ahmed Abdelwahab, and Gary Saulnier. ACT5 EIT System. In *21<sup>st</sup> International Conference on Biomedical Applications of Electrical Impedance Tomography (EIT2021)*, page 55, 2021.

Acknowledgements: SH was supported by the National Institute of Biomedical Imaging and Bioengineering of the National Institutes of Health under award number R21EB028064. JT and VK were supported by the Academy of Finland (Project 312434) and the Jane and Aatos Erkkö Foundation and Neurocenter Finland.

## An *in-vivo* comparison of source localization methods

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Electrical source imaging (ESI) aims at reconstructing the electrical brain activity from measurements of the electric field on the scalp. ESI is a key element in the analysis of EEG data, in both research and clinical settings.

In the last twenty years several algorithms have been applied for solving the ill-posed EEG inverse problem. Most of these popular methods can be derived within a Bayesian statistical framework in which all variables can be modelled as random variables with associated probability density functions (pdf) and the solution of the inverse problem is the posterior pdf for the unknown primary current distribution conditioned on the measurements. The different methods mainly differ from each other by the quality and quantity of a priori information they use in order to solve the EEG inverse problem. In this study [1] we validate and compare ten different ESI methods (wMNE, dSPM, sLORETA, eLORETA, LCMV, dipole fitting, RAP-MUSIC, MxNE, gamma map and Sesame) "in vivo", by exploiting a recently published EEG dataset [2] for which the ground truth is known. We compare the different inverse methods under multiple choices of input parameters, to assess the accuracy of the best reconstruction, as well as the impact of the parameters on the localization performance.

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## Nonlinear estimation of potential flow in background-oriented schlieren imaging

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In medical ultrasound applications the ultrasound field output needs to be assessed to ensure consistent, accurate, and safe treatment delivery [1]. However, existing ultrasound characterization techniques can be cumbersome to use, slow, and expensive.

Background-oriented schlieren imaging is a recently proposed optical imaging method for estimating projections of ultrasound fields [2, 3]. This method is based on observing deflection of light through a heterogeneous refractive index field induced by the ultrasound field. The deflections manifest as apparent perturbations in an imaged target. This can be formulated as a potential flow problem [4] and solved to estimate projection of the ultrasound field. Typically, the estimation approaches are derived using linear approximations, which can limit the applicability of the imaging approach to low amplitude ultrasound fields.

We propose a nonlinear potential flow model to alleviate limitations of approaches that linearize the problem [5]. The approach is formulated as a nonlinear regularized minimization problem. The proposed approach is compared to a linear approach by estimating projections of high-intensity ultrasound fields. The nonlinear approach is shown to outperform the linear one (Figure 1).

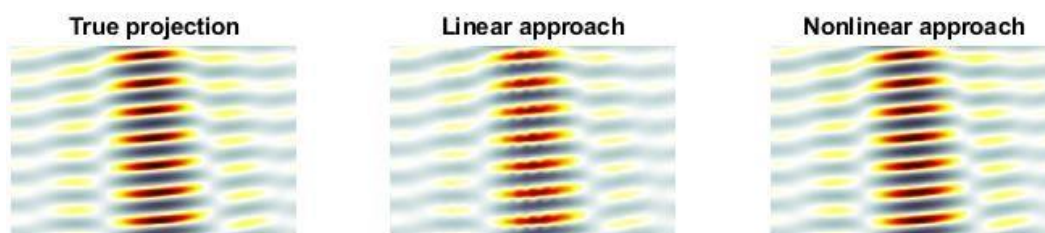


Figure 1: The true projection of an ultrasound field, estimated linear and nonlinear projections. Linear and nonlinear estimates have relative errors of 16.7% and 5.8% respectively.

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Acknowledgements: This work has been supported by the Academy of Finland (projects 314411, 336799 Center of Excellence in inverse Modelling and Imaging, and 320166 Flagship Programme Photonics Research and Innovation), the Instrumentarium Science Foundation, and the Vilho, Yrjö and Kalle Väisälä Foundation of Finnish Academy of Science and Letters.



## Learning a microlocal prior for limited-angle tomography

Siiri Rautio, Rashmi Murthy, Tatiana A. Bubba, Matti Lassas and Samuli Siltanen

Digital Breast Tomosynthesis (DBT) is a medical imaging technique for performing limited-angle reconstruction of the breast. Compared to traditional digital mammography, DBT has many advantages. Most importantly, instead of a single superimposed mammogram, DBT reconstructions are tomographic slice images. This cross-sectional information is useful in uncovering otherwise hidden masses and reducing the number of false positive cancer findings. However, the limited-angle imaging geometry leads to the reconstruction problem being severely ill-posed, resulting in stretching artefacts distorting the reconstruction.

We use the Primal-Dual Fixed-Point (PDFP) algorithm together with a complex wavelet-based prior to compute initial slice-by-slice reconstructions of simplified 3D breast phantoms. The PDFP algorithm enforces a non-negativity constraint, while the complex wavelets promote sparsity in the wavelet basis. To improve these initial reconstructions, we use deep learning to understand how the known boundaries extend into the unknown area. We train a neural network to fill in the missing wave front set in the complex wavelet coefficients, and use this information to construct the singular support the target. The skeleton of the singular support is then used as an overlay for the reconstruction. The features in the initial reconstruction are stretched along the central direction of projections, but with the overlay, it is possible to reveal the true form and extent of the features.

The slice-by-slice reconstructions are computed for vertical slices of the 3D phantom, stacked together to form a three-dimensional reconstruction, and sliced horizontally for the final result. For comparisons, we have computed reconstructions with unfiltered backprojection (UFBP), which is the traditional reconstruction technique for tomosynthesis.

# Sound speed uncertainty in Acousto-Electric Tomography

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June 2021

## Abstract

The goal in Acousto-Electric Tomography (AET) is to reconstruct an image of the unknown electric conductivity inside an object from boundary measurements of electrostatic currents and voltages collected while the object is penetrated by propagating ultrasound waves. This problem is a coupled-physics inverse problem. Accurate knowledge of the propagating ultrasound wave is usually assumed and required, but in practice tracking the propagating wave is hard due to inexact knowledge of the interior acoustic properties of the object. In this work, we model uncertainty in the sound speed of the acoustic wave, and formulate a suitable reconstruction method for the interior power density and conductivity. We also establish theoretical error bounds, and show that the suggested approach can be understood as a regularization strategy for the inverse problem. Finally, we numerically simulate the sound speed variations from a numerical breast tissue model, and computationally explore the effect of using an inaccurate sound speed on the error in reconstructions. Our results show that with reasonable uncertainty in the sound speed reliable reconstruction is still possible.

*Keywords:* Acousto-Electric Tomography, acousto-electric effect, Electrical Impedance Tomography, uncertainty quantification, hybrid data tomography, variable sound speed, coupled-physics imaging, inverse problems, medical imaging

**MSC2010:** 35R30, 65N21

## L1 vs. L2 norm fitting in optimizing focal multi-channel tES stimulation: linear programming vs. weighted and re-weighted least-squares

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In this numerical simulation study, we consider the task to optimize stimulation currents in a multichannel version of the transcranial electrical stimulation (MC-tES), a noninvasive brain stimulation (NIBS) method applied for stimulating neuronal activity under the influence of low-intensity currents. We aim at finding current patterns that would optimize a L1 norm fit between a given focal target current distribution and a current distribution while minimizing non-zero currents in the pattern with specific conditions such as adjusting total dose or extrinsic physiological conditions. We present an L1 norm regularized linear programming (LP) approach that can be motivated to find a focal current distribution. The results obtained using an electrode montage of 2-, 3-, 5- and 8-channels suggest that our approach can be implemented. The alternative current pattern found via our LP approach present a set of advantages when compared to weighted Tikhonov Least-Squares (TLS) and L1 regularized least-squares (L1LS) methods.

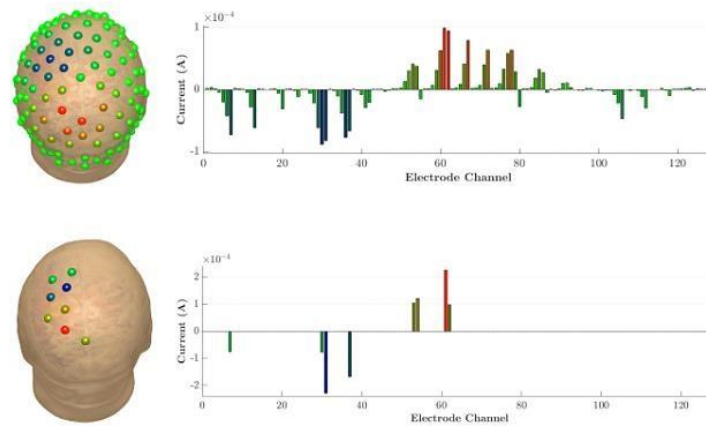


Figure 1: Pre-optimized 128-channel (top) and their potential values before limiting to the highest current value contributors for an 8-channel version montage (bot).

- [1] Fernandez-Corazza, M., S. Turovets, and C. H. Muravchik (2020). "Unification of optimal targeting methods in transcranial electrical stimulation". In: *Neuroimage* 209, p. 116403
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Acknowledgements: FGP, AR and SP were supported by the Academy of Finland Center of Excellence in Inverse Modelling and Imaging 2018--2025, DAAD project (334465) and by the ERA PerMed project PerEpi (344712). AR was also supported by the Alfred Kordelini Foundation.

## **$\ell_1$ -Reweight Hierarchical Bayesian Method for EEG Source Localization via Randomized Multiresolution Scanning**

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In the pursuit of reconstruction of focal neural activity,  $\ell_1$ -penalization for the EEG norm minimization problem is a natural choice against the  $\ell_2$ -penalization. In order to obtain such a hierarchical Bayesian model, one needs to be able to choose penalization meaningfully, meaning that penalization weighting comes from an appropriate prior model. In this presentation, the prior model has been selected to be the *conditionally exponential distribution* that expands the reweighting to any  $\ell_p$ -norm. Using this as an advantage, we can straightforwardly compare reconstructions obtained via  $\ell_1$ - and  $\ell_2$ -reweighting in cortical and sub-cortical levels. To enhance the detectability in sub-cortical regions, the randomized multiresolutions scanning (RAMUS) method<sup>[1]</sup> is used.

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## Contact adapting electrode model for electrical impedance tomography

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Practical electrical impedance tomography reconstruction algorithms are usually based on Complete Electrode Model (CEM) as the forward model. However, if the electrode locations are imprecisely known, earlier research has shown that such algorithms are susceptible to artifacts in the domain reconstruction. An error as small as a couple millimeters for the electrode locations can have a substantial effect in a torso-sized object of interest. In this presentation, we describe an alternative electrode model which can mitigate these reconstruction artifacts significantly.

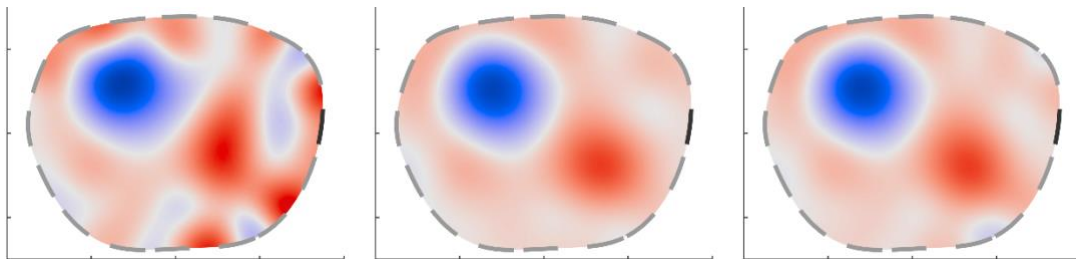


Figure 1: Reconstructions of a real-world water tank measurement with erroneous electrode locations, left being the result of standard Complete Electrode Model while the center and right figures are variants of our approach.

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Acknowledgements: The work by JD was supported by the Institut Français de Finlande, the Embassy of France in Finland, the French Ministry of Higher Education, Research and Innovation, the Finnish Society of Sciences and Letters and the Finnish Academy of Science and Letters (2019 Maupertuis Programme), the work by NH and TK was supported by the Academy of Finland (decision 312124), and the work by TV was supported by the Academy of Finland (decisions 314701 and 320022).

## Inverse Approach for Identifying Intrinsic Electromagnetic Material Parameters of Ferrite Cores

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The electromagnetic material parameters, i.e., conductivity  $\sigma$ , permittivity  $\varepsilon$ , and permeability  $\mu$  of Mn-Zn ferrite cores should be accurately known to analyze energy losses in high-frequency inductors and transformers. Direct measurement of these parameters is not an easy task due to the granular structure of the material and the non-uniform field distribution in the sample at high frequency [1]-[2]. Thus, inverse approaches should be employed for identifying such material parameters.

In this work, an inverse approach is presented for extracting the intrinsic  $\sigma$ ,  $\varepsilon$ , and  $\mu$  of the grains (subscript g) and grain boundaries (subscript gb) for Mn-Zn ferrite cores. A 2-D finite element (FE) model is developed for the forward problem in which the full-wave electromagnetic field in the ferrite core is solved in the grain-scale microstructure. From Maxwell equations, the governing equation for the electric field strength  $\mathbf{E}(r, z) = E_r(r, z)\hat{\mathbf{r}} + E_z(r, z)\hat{\mathbf{z}}$  in a 2-D cross section of a disk core with radius  $R$  and height  $d$  placed between two electrodes is

$$\nabla \times \left( (j\omega\mu)^{-1} \nabla \times \mathbf{E}(r, z) \right) + (\sigma + j\omega\varepsilon) \mathbf{E}(r, z) = 0, \quad (1)$$

$$E_r(r, 0) = E_r(r, d) = 0 \quad \forall r \in [0, R] \quad \text{and} \quad E_z(R, z) = \frac{U}{d} \quad \forall z \in [0, d] \quad (2)$$

where  $U$  is the supplied voltage and  $\omega$  is its angular frequency.

Let  $\Lambda = [\sigma_g, \sigma_{gb}, \varepsilon_g, \varepsilon_{gb}, \mu_g, \mu_{gb}]$  be a vector containing the material parameters of the grains and grain boundaries. The impedance  $Z_c(\omega, \Lambda)$  is calculated as

$$Z_c(\omega, \Lambda) = \frac{|U|^2}{\int \left( (\sigma + j\omega\varepsilon) \|\mathbf{E}\|^2 + (j\omega\mu)^{-1} \|\nabla \times \mathbf{E}\|^2 \right) d\Omega}. \quad (3)$$

The inverse problem can be formulated as an optimization problem

$$\Lambda = \arg \min_{\Lambda} \sum_{i=1}^n |Z(\omega_i) - Z_c(\omega_i, \Lambda)|^2, \quad (4)$$

where  $Z(\omega_i)$  are the impedances measured at the frequencies  $\omega_i, i = 1, \dots, n$  of interest. Iterative techniques, e.g., conjugate direction methods, in combination with the forward FE model and impedance measurements need to be used for solving (4) [3].

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- [3] J. Mueller and S. Siltanen, *Linear and Nonlinear Inverse Problems with Practical Applications*, SIAM (Computational Science & Engineering), 2012.

## Material-separating regularizer for multi-energy X-ray tomography

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Dual-energy X-ray tomography is considered in a context where the target under imaging consists of two distinct materials. The materials are assumed to be possibly intertwined in space, but at any given location there is only one material present. Further, two X-ray energies are chosen so that there is a clear difference in the spectral dependence of the attenuation coefficients of the two materials. A novel regularizer is presented for the inverse problem of reconstructing separate tomographic images for the two materials. A combination of two things, (a) non-negativity constraint, and (b) penalty term containing the inner product between the two material images, promotes the presence of at most one material in a given pixel. A preconditioned interior point method is derived for the minimization of the regularization functional.

Numerical tests with digital phantoms suggest that the new algorithm outperforms the baseline method, Joint Total Variation regularization, in terms of correctly material-characterized pixels. While the method is tested only in a two-dimensional setting with two materials and two energies, the approach readily generalizes to three dimensions and more materials. The number of materials just needs to match the number of energies used in imaging.

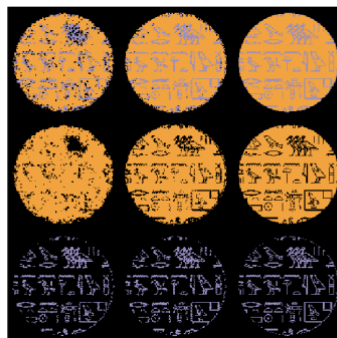


Figure 1: Segmentation results. The first row shows both materials of the phantom, the second row shows only material 1 and the third row shows only material 2. The first column shows JTV segmentations, the second shows IP segmentations and the third is the ground truth.

- [1] Jussi Toivanen, Alexander Meaney, Samuli Siltanen, and Ville Kolehmainen. Joint reconstruction in low dose multi-energy CT. *Inverse Problems & Imaging*
- [2] Jennifer L Mueller and Samuli Siltanen. *Linear and nonlinear inverse problems with practical applications*. SIAM, 2012.
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Acknowledgements: This work was supported by DOMAST.

# Deterministic and statistical characterisation of poroelastic media from sound absorption measurements

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In this work, a framework for estimating the properties of poroelastic media using sound absorption measurements is proposed. A one-dimensional Biot-Johnson-Champoux-Allard model [1] is chosen as the underlying model, where a total of 9 constants are of interest: five transport parameters, two elastic constants, the mass density and the sample thickness.

In the studied case, these constants are estimated from frequency-dependent sound absorption measurements. An over-determination of the inverse problem is proposed by combining multiple measurements, obtained by placing the sample in a two-microphone impedance tube under different loading conditions (Figs. (a-c)). The setup is modelled analytically using the transfer matrix method [1], thus enabling low-resource iterative computations. The model inversion procedure consists of two steps. First, an initial deterministic estimate is obtained using an incremental search [2] whose purpose is to overcome local minima. A statistical inversion is then performed and provides refined point estimates, uncertainty ranges and parameter correlations (Fig. (d)).

The method is demonstrated on a sample of melamine foam [3]. It is shown that the retrieved uncertainty on the material parameters is reduced by combining multiple sound absorption measurements.

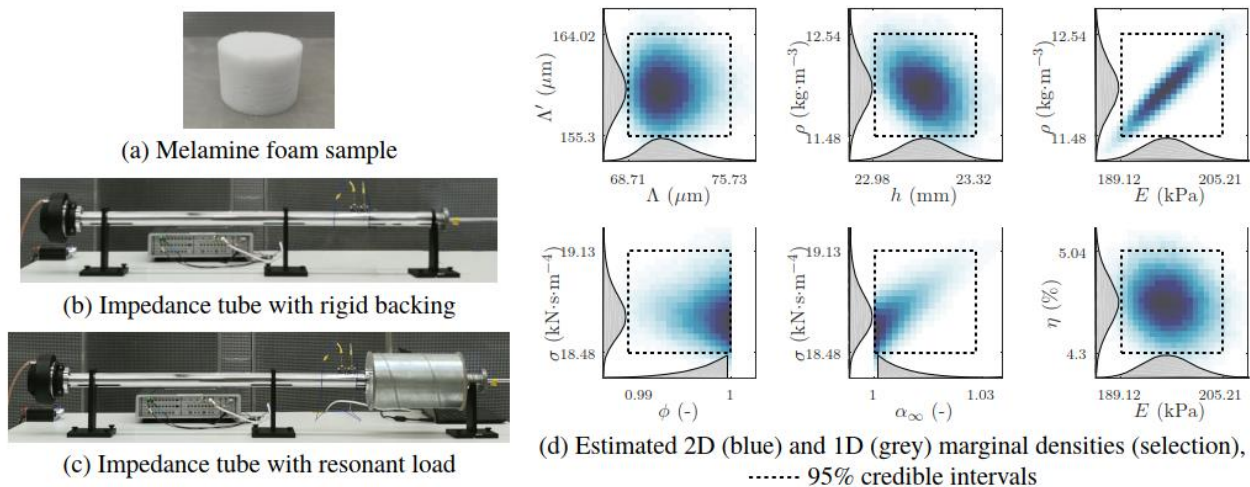


Figure: Characterisation of a sample of melamine foam [3]

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## Model-based estimation of forest characteristics using remotely sensed data.

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Remote sensing is increasingly used in collecting data for forest inventories. (1) In aerial laser scanning, a laser scanner is installed to airplane or a drone and the device is used to collect information about the forest canopy below. Individual tree detection can further be done using the laser point cloud to extract individual tree crowns and estimate the tree heights and tree crown size and shape. (2) In terrestrial laser scanning, a scanner is placed on a tripod and the surrounding forest is scanned for measurements of tree stems. Individual tree stems can be detected from the point cloud and used for estimation of tree DBH and other tree characteristics. (3) Laser scanners can also be used from satellites. In those cases, laser footprints are too large for detecting individual trees.

In this talk, I will discuss different models for estimation of stand density and size distribution of trees in all three abovementioned cases. In cases (1) and (2) the detected trees can be ordered according to their shortest distance to the sensor. Because of this hierarchical ordering of trees, trees that are “earlier” in the hierarchy can cause “latter” trees to remain undetected because they are located in the hidden area or sector formed by the earlier trees [1,2]. For potential application in case (3), we discuss a statistical model for the marginal distribution of canopy height, based on stand density, tree crown shape and internal structure and spatial pattern of tree locations [3].

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Acknowledgements: This work was partially supported by Academy of Finland Project 310073 and the flagship program (UNITE, Decision Number 337655).

## Quantifying TLS Data and Tree Reconstruction Uncertainty

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To study and quantify important tree properties, such as tree diameter, terrestrial laser scanning can be used to reconstruct tree geometry. In forestry applications, this data can contain a large amount of noise, however [1, 2]. Uncertainties in vegetation volume and biomass account for a large part of the uncertainty of terrestrial carbon cycle models [3]. Biomass is frequently calculated with allometric equations using the tree diameter [4, 5], meaning that errors in the tree diameter propagate into quantities used in environmental modelling.

This research regards a method to estimate the uncertainty within the laser scanning data and propagate it to the estimated tree geometry through Monte Carlo sampling. To model the uncertainty of the data, continuous probability distributions of different magnitudes and direction are used, as shown in Figure 1. Additionally, this uncertainty information is used to improve the accuracy of shape fitting using the maximum likelihood approach.

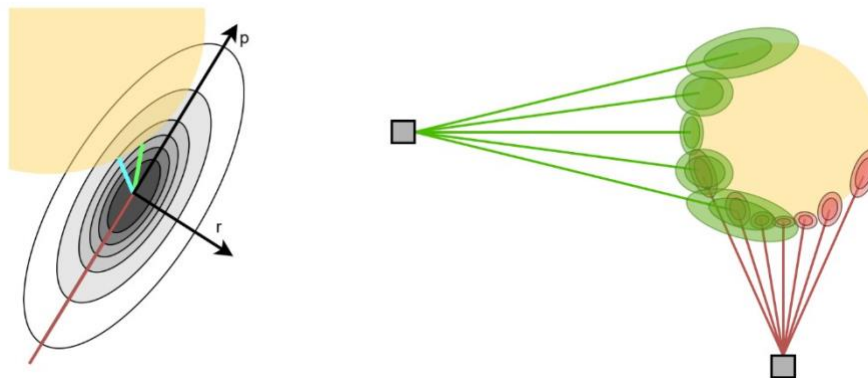


Figure 1: TLS data uncertainty. **(a)** The blue and green lines are the closest and highest likelihood intersections with the yellow circle estimate respectively. **(b)** The green and red points have different uncertainty magnitudes and direction.

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- [5] Paul, K. I., Roxburgh, S. H., England, J. R., Ritson, P., Hobbs, T., Brooksbank, K., ... & Rose, B. (2013). Development and testing of allometric equations for estimating above-ground biomass of mixed-species environmental plantings. *Forest Ecology and Management*, 310, 483-494.

Acknowledgements: This work was supported by Academy of Finland Centre of Excellence of Inverse Modelling and Imaging.



## Sawing Optimization, a Tandem Forest values project

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The main steps of the sawing optimization process we consider in this project consist in getting accurate CT images from limited sequentially obtained projection data, segmenting the reconstructed trunk, and finally optimizing the cutting parameters.

The goal of all the stakeholders involved in the sawing processes is to produce the highest possible quality boards from each log, while minimizing the waste of time and resources. Among the most important factors determining the quality of a panel, and consequently also its value, one can certainly find the resistance to bending and the absence of foreign objects like metallic nails. It is therefore important to avoid the presence of wood knots on the lateral edge of the boards as it can considerably compromise the resistance to bending of this latter.

In this talk we will present the imaging pipeline and various approaches to obtain a reconstruction of the trunks from classic variational approaches to learned methods. We then compare precision of the reconstructions in terms of precision of the subsequently obtained segmentation with respect to an accurate reconstruction from 360 angles.

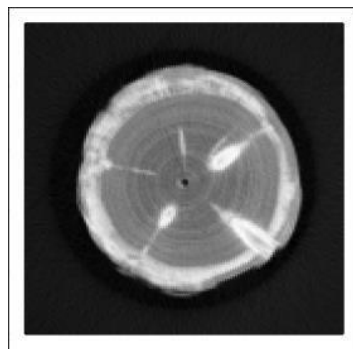


Figure 1: Example of trunk section



## Analyzing nitrogen oxides to carbon dioxide emission ratios from space: A case study of Matimba Power Station in South Africa

Janne Hakkarainen<sup>1</sup>, Monika Szelaġ<sup>1</sup>, Iolanda Ialongo<sup>1</sup>, Christian Retscher<sup>2</sup>, Tomohiro Oda<sup>3</sup>, and David Crisp<sup>4</sup>

<sup>1</sup> Finnish Meteorological Institute

<sup>2</sup> European Space Agency

<sup>3</sup> Universities Space Research Association (USRA)

<sup>4</sup> JPL/Caltech

Using satellite data for estimating carbon dioxide (CO<sub>2</sub>) emissions from local anthropogenic sources such as power plants and cities has become increasingly important since the Paris Agreement was adopted in 2015. For example, the very first study that estimated CO<sub>2</sub> emissions from individual power plants was published as late as 2017, although, simulation-based studies had been made also earlier. Here we describe the inverse problem related to emission estimation from satellite observations. In particular, we describe a new methodology for deriving source-specific emission ratios of nitrogen oxides (NO<sub>x</sub>) to carbon dioxide (CO<sub>2</sub>) from space-based TROPOMI and Orbiting Carbon Observatory-2 (OCO-2) observations. The approach is based on scaling the observed ratio along the OCO-2 track with simulated data to obtain the NO<sub>x</sub>-to-CO<sub>2</sub> emission ratio at the source. We analyze fourteen TROPOMI/OCO-2 collocations from near the Matimba coal-fired power station in South Africa. We obtain a mean NO<sub>x</sub>-to-CO<sub>2</sub> emission ratio of  $2.6 \times 10^{-3}$  and standard deviation of  $0.6 \times 10^{-3}$  (or 23%). When applied to NO<sub>x</sub> emission estimates derived from TROPOMI data, we obtain annual CO<sub>2</sub> emissions of about 60 kt/d with standard deviation of 20 kt/d (or 33%). These values are consistent with existing inventories such as the Open-source Data Inventory for Anthropogenic CO<sub>2</sub> (ODIAC). The proposed method will also work ideally for new and upcoming satellite observations systems such as OCO-3, CO2M and GOSAT-GW.

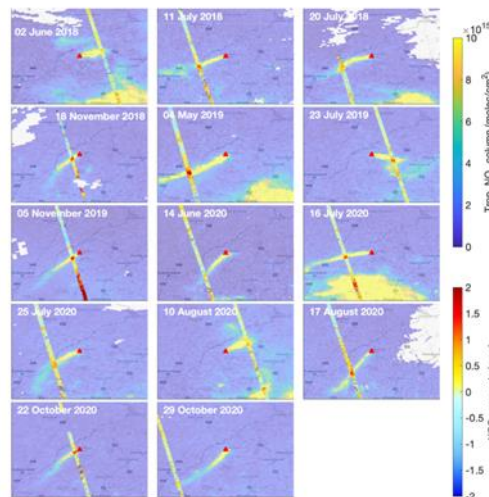


Figure 1: OCO-2 and TROPOMI observations near Matimba power station (red triangle) in South Africa between May 2018 and November 2020. Image: Hakkarainen et al. 2021. CC BY 4.0.

[1] Janne Hakkarainen, Monika E. Szelaġ, Iolanda Ialongo, Christian Retscher, Tomohiro Oda, and David Crisp: Analyzing nitrogen oxides to carbon dioxide emission ratios from space: A case study of Matimba Power Station in South Africa, Atmospheric Environment: X, Volume 10, <https://doi.org/10.1016/j.aeaoa.2021.100110>, 2021.

## Stability of the Gel'fand's inverse boundary spectral problem via the unique continuation

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The Gel'fand's inverse boundary spectral problem concerns determining the geometry of a compact manifold with boundary from the boundary spectral data for the Laplacian. We show that this problem has a stable solution with quantitative stability estimates in a class of Riemannian manifolds with bounded geometry. More precisely, we show that finitely many Neumann eigenvalues and the boundary values of the corresponding eigenfunctions, known up to small errors, determine a metric space that is close to the manifold in the Gromov-Hausdorff sense. This result is based on an explicit estimate on the stability of the unique continuation for the wave operator from a subset of the manifold boundary.

[1] D. Burago, S. Ivanov, M. Lassas, and J. Lu. Stability of the Gel'fand inverse boundary problem via the unique continuation. arXiv:2012.04435.

# Reconstructing anisotropic conductivities on 2D Riemannian manifolds from power densities

Kim Knudsen<sup>1</sup>, Steen Markvorsen<sup>1</sup>, Hjørdis Schlüter<sup>1</sup>

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We present an approach to reconstruct the anisotropic electrical conductivity on a compact 2-dimensional manifold with genus 0 and non-empty smooth boundary. This is based on knowledge from interior power densities. The key novelties of the presented work is the generalization from simply connected 2-dimensional Euclidean domains to compact 2-dimensional Riemannian manifolds with boundary. The reconstruction procedure is visualized for a finite cylinder with two circular boundary components. Furthermore, it is discussed whether these results can be generalized to 2-dimensional Riemannian manifolds with higher genus.

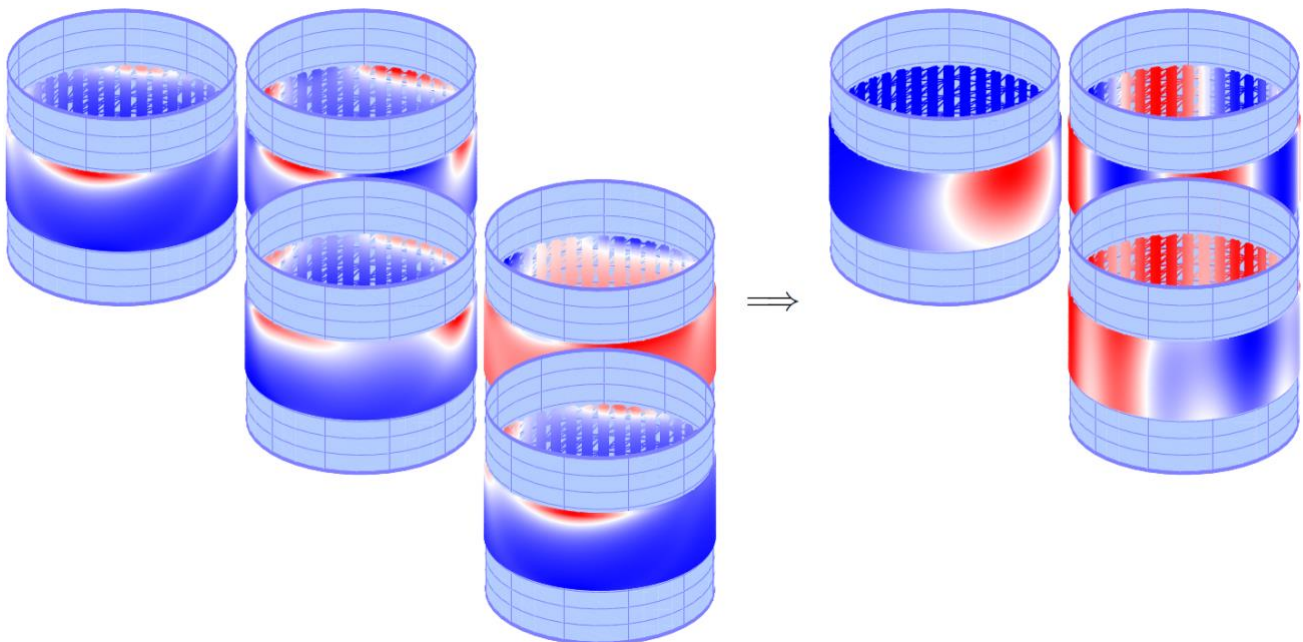


Figure: Reconstruction procedure on a cylinder: Illustration of the power density measurements (left) used for reconstructing an anisotropic conductivity (right) determined by three scalar functions with respect to the canonical basis of the cylinder.

## A compound Krylov subspace method for parametric linear systems

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When solving inverse problems, one sometimes needs to solve parameter-dependent partial differential equations using the finite element method (FEM). In this case, one ends up with a parametrized system of linear equations.

Our work concerns efficient solution of the following linear system: Let  $b \in \mathbb{R}^n$ , parameter set  $S \subset \mathbb{R}^s$  and assume  $A: \mathbb{R}^s \rightarrow \mathbb{R}^{n \times n}$  is a linear matrix-valued function such that  $A(S)$  is a real, symmetric and positive definite matrix. Find  $x(\sigma) \in \mathbb{R}^n$  satisfying

$$A(\sigma)x(\sigma) = b$$

for multiple values of the parameter  $\sigma \in S$ .

I will introduce a novel *compound Krylov subspace* method which is a method to efficiently compute approximate solutions to the above parametric linear system. The method is inspired by the conjugate gradient method which is an iteration for finding a sequence of approximate solutions to non-parametric linear systems with symmetric positive definite matrices. Our method is a reduced basis method: we find a smaller linear system which has a solution that approximates the original solution.

I will also present numerical examples related to solving the conductivity equation using FEM in a grid with a varying conductivity (Fig. 1). The parameter  $\sigma$  is now a vector containing the conductivities of each colored subdomain. Using our method, we can solve this problem quickly for multiple values of the parameter vector  $\sigma$ .

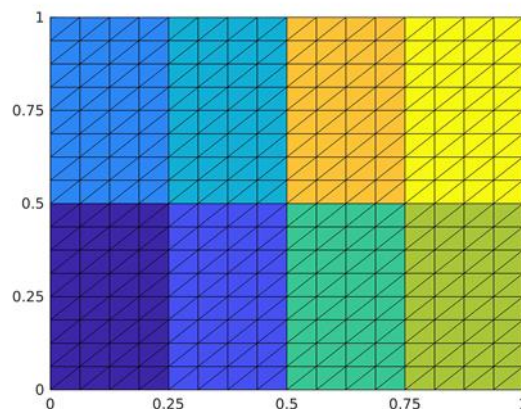


Figure 1: A 2x4 grid where each colored subdomain may have a different conductivity.

## Quasi-Monte Carlo methods for optimal control problems subject to time-dependent PDE constraints under uncertainty

Philipp A. Guth<sup>1</sup>, Vesa Kaarnioja<sup>2</sup>, Frances Y. Kuo<sup>3</sup>, Claudia Schillings<sup>1</sup>, Ian H. Sloan<sup>3</sup>

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<sup>2</sup> School of Engineering Science, LUT University, P.O. Box 20, 53851 Lappeenranta, Finland.

<sup>3</sup> School of Mathematics and Statistics, University of New South Wales, Sydney NSW 2052, Australia.

Modern quasi-Monte Carlo (QMC) methods are based on tailoring specially designed cubature rules for high-dimensional integration problems—an especially prominent application within the field of uncertainty quantification is computing the statistical response of PDEs with random or uncertain coefficients. In this work, we study the application of a tailored QMC method to a class of optimal control problems subject to time-dependent PDE constraints under uncertainty: the state in our setting is the solution of a parabolic PDE with a random thermal diffusion coefficient, steered by a control function.

To account for the presence of uncertainty in the optimal control problem, the objective function is composed with a risk measure. We consider risk measures involving high-dimensional integrals over the stochastic variables such as the (nonlinear) entropic risk measure. The high-dimensional integrals are computed numerically using QMC and, under moderate assumptions on the input random field, the error rate is shown to be essentially linear independently of the stochastic dimension of the problem—and thereby superior to ordinary Monte Carlo methods. Numerical results are presented to assess the effectiveness of our method.



# Causal coupling inference from multivariate time series based on ordinal partition transition networks

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Synchronization is broadly defined as the tendency for several subsystems within a complex system to behave in a similar manner. For example, in neuroscience hyper-synchronization among neuronal populations can give rise to disorders such as epilepsy. Detecting synchronization among subsystems is fundamental to understanding the mechanisms subserving the complex system. However, in order to design intervention strategies, make predictions, optimize and control the dynamics of complex systems, we need to identify the causal relationships (i.e. **what causes what**) among the subsystems.

This is a challenging yet crucial problem in many fields of science, including epidemiology, climatology, genomics, economics and neuroscience, to mention only a few. Recent studies have demonstrated that ordinal partition transition networks (OPTNs) allow inferring the causality between two dynamical systems. In this work, we generalize this concept and propose a new method to study causal relations among multiple dynamical systems in a complex system using observational data. By applying this method to numerical simulations of coupled linear stochastic processes as well as two examples of interacting nonlinear dynamical systems (coupled Lorenz systems and a network of neural mass models), we demonstrate that our approach can reliably identify the direction of interactions and the associated coupling delays. Finally, we study real-world observational microelectrode array electrophysiology data from rodent brain slices to identify the causal coupling structures (see Figure 1) underlying epileptiform activity. Our results, both from simulations and real-world data, suggest that OPTNs can provide a complementary and robust approach to infer causal effect networks from multivariate observational data.

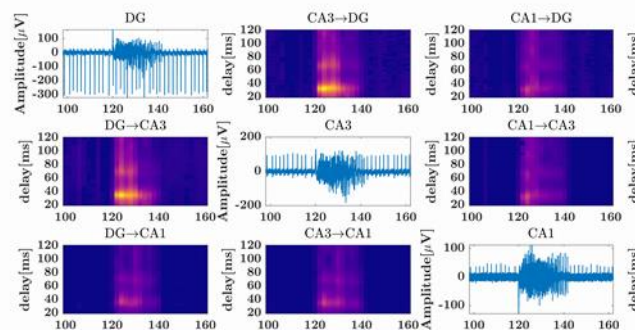


Figure 1: Causal coupling structure (causal strength color-coded ; bright – high, dark-low) and associated delays underlying epileptic activity among different regions of hippocampus ( Dentate gyrus (DG), Cornu Ammonis (CA)) of rodent brain slice. The X-axis represents time in seconds.

Acknowledgements: This project has received funding from the European Union’s Horizon 2020 research and innovation programme FETPROACT-01-2018 (RIA) awarded to the project Hybrid Enhanced Regenerative Medicine Systems (HERMES) under grant agreement No 824164.

## Real-time Ionospheric Imaging of Scintillation from Limited Data with Parallel Kalman Filters

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Ionospheric scintillation is observed as rapid changes in the amplitude and phase of radio signals (that are used for GPS navigation, for example) that pass through the ionosphere. These rapid changes are caused by small-scale irregularities in ionospheric medium (e.g. plasma bubbles). Scintillation is observed primarily in the equatorial and auroral regions. Monitoring of the ionospheric scintillation is crucial for evaluating the robustness of satellite-based communication and navigation systems. So far, limited amount of data (and limited number of ground monitoring stations) available have hindered the resolution of ionospheric imaging.

In this talk, I will present a method to create two-dimensional ionospheric images of scintillation with high spatio-temporal resolution using Kalman filtering. To cope with the limited amount of data, I will introduce an extra set of equations with connectivity information stemming from assumptions about the spatial distribution of the scintillation activity on the ionospheric shell to augment the observation model. To reduce the uncertainty related to the tuning parameters of the proposed augmented observation model, I will use an ensemble of Kalman filters running in parallel. In particular, each member of the ensemble is modeled with a different realization of the tuning parameter (for the connectivity prior) selected from a predetermined set of values. The output images are weighted averages of the state estimates of the individual filters. The weights are scalar quantities which are estimated (on-line) based on the performance of the individual filters when compared to control measurements. Finally, I will demonstrate the proposed method by rendering two dimensional ionospheric images of scintillation activity at 350 km over South America with temporal resolution of one minute.

- [1] A. Koulouri. Real-time ionospheric imaging of S4 scintillation activity from Limited Data with Parallel Kalman Filters and Smoothness, IEEE Transactions on Geoscience and Remote Sensing (Accepted)
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Acknowledgements: I was supported by the Academy of Finland Postdoctoral Researcher program (No 326454).

# Bayesian inverse problems, prior modelling and algorithms for posterior sampling

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Bayesian inverse problems provide a coherent mathematical and algorithmic framework that enables researchers to combine mathematical models with data. The ability to solve such inverse problems depends crucially on the efficient calculation of quantities relating to the posterior distribution, which itself requires the solution of high dimensional optimization and sampling problems. In this talk, we will study different algorithms for efficient sampling from the posterior distribution under two different prior modelling paradigms. In the first one, we use specific non-smooth functions, such as for example the total variation norm, to model the prior. The main computational challenge in this case is the non-smoothness of the prior which leads to “stiffness” for the corresponding stochastic differential equations that need to be discretised to perform sampling. We address this issue by using tailored stochastic numerical integrators, known as stochastic orthogonal Runge-Kutta Chebyshev (S-ROCK) methods, and show that the corresponding algorithms are able to outperform the current state of the art methods. In the second modelling paradigm, the prior knowledge available is given in the form of training examples and we use machine learning techniques to learn an analytic representation for the prior. The main computational challenge in this case is that the corresponding posterior distribution becomes multimodal which results in a challenging sampling problem since standard Markov Chain Monte Carlo methods (MCMC) can get stuck in different local maxima of the posterior distribution. We address this issue, by using specifically designed MCMC methods and exhibit numerically that this “data-driven” approach improves the performance in a number of different imaging tasks, such as image denoising and image deblurring.





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