Algebraic degree of polynomial optimization

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Abstract

We study an optimization problem with the feasible set X defined by polynomials and whose parametric objective function f_u is gradient-solvable with respect to parametric data u. This class includes common objective functions such as the squared Euclidean distance and the log-likelihood function, but also the p-th power of the p-norm for an integer $p \geq 2$.

For Euclidean distance (ED) optimization and maximum likelihood (ML) optimization, a prominent role is played by the ED and ML correspondence, respectively. To our generalized optimization problem we attach an optimization correspondence $\mathcal{F}_X$ which is the set of points $(x, u) \in \mathbb{C}^n \times \mathbb{C}^n$ such that $x \in X$ is critical for f_u. This leads to the notion of algebraic degree of optimization on X. Over general data points $u \in \mathbb{C}^n$, the second projection $\pi_2 : \mathcal{F}_X \to \mathbb{C}^n$ has finite fibers $\pi_2^{-1}(u)$ of constant cardinality. The algebraic degree of X with respect to the objective function f_u is the cardinality of the fiber $\pi_2^{-1}(u)$ over a general data point u. The algebraic degree of optimization measures the algebraic complexity of the optimal solution. We apply these results to p-norm optimization, and define the p-norm distance degree of X, which coincides with the ED degree of X for $p = 2$.

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