Computing in AF C*-algebras via algebraic logic

DANIELE MUNDICI
Department of Mathematics and Computer Science
University of Florence
Florence, Italy
daniele.mundici@unifi.it
http://web.math.unifi.it/users/mundici/

ABSTRACT

An *AF(C*)-algebra A* is the norm-closure of the union of an ascending sequence of finite-dimensional C*-algebras. Elliott proved that the unital dimension (Grothendieck) group $K_0(A)$ uniquely determines A up to isomorphism. AF-algebras with lattice-ordered K_0 (for short, *AFl-algebras*) have a preeminent role in the AF-algebraic literature. Let Γ be the categorical equivalence between unital abelian l-groups and MV-algebras. The Elliott classifier $E(A)=\Gamma(K_0(A))$ of any AFl-algebra A has the structure of a countable MV-algebra. All countable MV-algebras arise as $E(A)$ for some AFl-algebra A. $E(A)$ uniquely determines A up to isomorphism. Elements of $E(A)$ are Murray-von Neumann equivalence classes of projections. Since $E(A)$ is the Lindenbaum algebra of some theory in Łukasiewicz infinite-valued logic $Ł_\infty$, every $Ł_\infty$-formula φ naturally codes a Murray-von Neumann equivalence class $[p]=[p]_\varphi$ of projections of A. The deductive-algorithmic machinery of $Ł_\infty$ can be applied to decide problems on the projections of A. For instance: Is $[p]$ the class of a central projection of A? Does $[p]$ precede $[q]$ in the Murray-von Neumann order of A? Is $[p]=0$? We construct polytime reductions among these problems in any fixed AFl-algebra. The complexity of all these problems turns out to be polytime for many relevant AFl-algebras in the literature, including the Behncke-Leptin algebras $\mathcal{A}_{m,n}$, the CAR algebra, the Farey-Stern-Brocot algebra, Glimm's universal UHF algebra, and every Effros-Shen algebra $\mathfrak{F}_\theta$ for θ a real algebraic integer, or $\theta = 1/e$.

References

G.A. Elliott, On the Classification of Inductive Limits of Sequences of Semisimple Finite-Dimensional Algebras, *J. Algebra*, 38 (1976) 29-44. https://doi.org/10.1016/0021-8693(76)90242-8

D.Mundici, Word problems in Elliott monoids, *Advances in Mathematics,* 335 (2018) 343-371. https://doi.org/10.1016/j.aim.2018.07.015

D.Mundici, Bratteli diagrams via the De Concini-Procesi theorem, *Communications in Contemporary Mathematics,* 23 Issue No. 7 (2021), Article No. 2050073. https://doi.org/10.1142/S021919972050073X